Last Name: ______________________ First Name: ______________________

Physics 101 Fall 2002: Test 1—Free Response and Instructions

- **Print your LAST and FIRST name on the front of your blue book, on this question sheet, the multiple-choice question sheet and the multiple-choice answer sheet.**
- **TIME ALLOWED 90 MINUTES**
- **The test consists of two free-response questions and ten multiple-choice questions.**
- **The test is graded on a scale of 100 points; the first free-response question accounts for 35 points, the second for 35 points and the multiple-choice questions account for 30 points.**
- **Answer the two free-response questions in your blue book. Answer the multiple-choice questions by marking a dark X in the appropriate column and row in the table on the multiple-choice answer sheet.**
- **Consult no books or notes of any kind. You may use a hand-held calculator in non-graphing, non-programmed mode.**
- **Do NOT take test materials outside of the class at any time. Return this question sheet along with your blue book and multiple-choice question sheet.**
- **Write and sign the Pledge on the front of your blue book.**

Show your work for the free-response problems, including neat and clearly labelled figures, in your blue book. Answers without explanation (even correct answers) will not be given credit.

---

1. **The people on train A are a little crazy. At $t = 0$, train A is traveling down a long, straight track at a speed $v_t = 30$ m/s. The train A is $L = 0.40$ km long and the last car at the very end of the train has a cannon mounted on it. Neglect the height of the cannon with respect to the ground. Unbelievably, the cannon is aimed at an angle $\theta_0 = 50°$ above the horizontal and in the direction of the train's motion. At $t = 0$, the cannon fires a ball at a speed of $v_b = 55$ m/s relative to the cannon. The firing of the cannon does not effect the train's speed; i.e., neglect recoil. Assume $g = 9.8$ m/s^2. The conductor of train A has to make a quick decision.**

   (a) **Assuming no air resistance, at what time will the ball land?**

   (b) **Where does the ball hit the track relative to where it was launched? If the conductor does not change the train's speed, where will the front and back of the train be at the time the ball lands?**

   (c) **If the conductor chooses to put on the brakes at time $t = 0$, what acceleration is required so that the train just stops before meeting the crater left by the ball hitting the track?**

   (d) **Draw a graph of the horizontal position versus time for the ball, the front of the train, and the back of the train for the case that the train continues with constant velocity and for the case that it just stops before it reaches the crater. On your graph, indicate the initial and final positions of the ball, the front of the train, and the back of the train.**

2. Consider two boxes being pulled by a cord across a floor with friction. The lower box has mass m_1 and a coefficient of kinetic friction of μ_k with the floor. The upper box has mass m_2 and a coefficient of static friction of μ_s with the lower box. The cord makes an angle of θ with the horizontal. Assume that the angle θ is sufficiently small that m_1 remains in contact with the floor.

First, assume the tension is large enough to make the lower box move.

(a) Draw a free body diagram showing all the forces acting on each box and give a brief description of each force.

(b) Apply Newton's second law in component form to each block and write down the resulting equations.

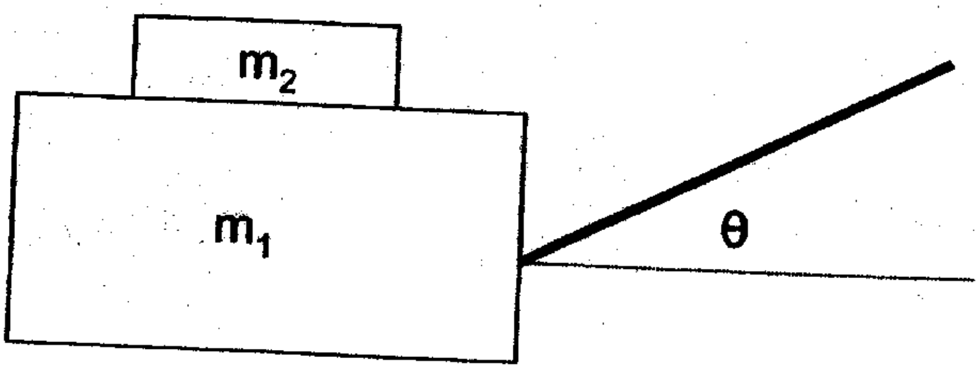
For the next part of the question, assume the boxes are pulled across the floor at constant velocity.

(c) Find the tension in the cord in terms of some or all of m_1, m_2, μ_k, μ_s and g.

For the next parts of the question, the tension is increased so that the boxes accelerate.

(d) What is the maximum acceleration for which the little box m_2 does not move with respect to the big box m_1?

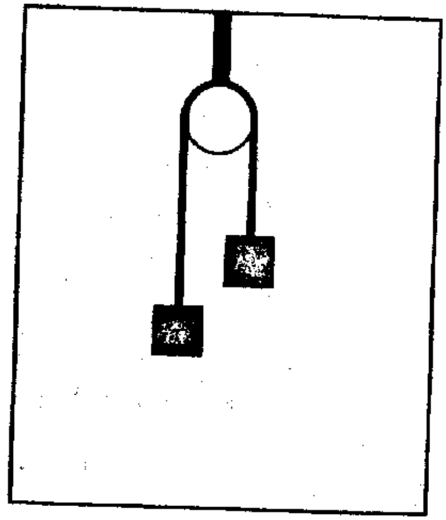
(e) What is the tension in the rope for this case expressed in terms of some or all of m_1, m_2, μ_s, μ_k and g?



Last Name: ______________________ First Name: ______________________

Physics 101 Fall 2002: Test 1—Multiple-Choice Questions

For questions 1-2: Two 5-kg masses are hung over a pulley. The pulley is suspended from the roof of an elevator. Use $g = 10\ \mathrm{m/s^2}$.



1. If the elevator is moving up at a constant velocity of 2 m/s, what is the tension in the string?

   (a) 25 N

   (b) 50 N

   (c) 70 N

   (d) 100 N

   (e) 140 N

2. If the elevator is accelerating down at $2\ \mathrm{m/s^2}$, what is the tension in the string?

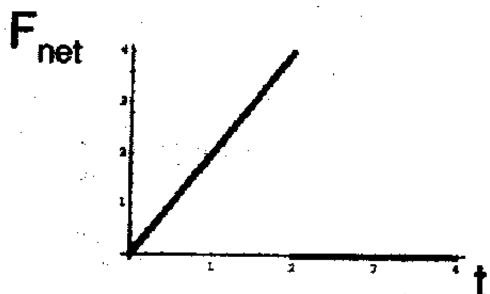
   (a) 10 N

   (b) 30 N

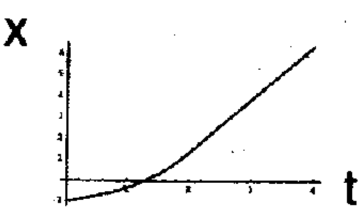
   (c) 40 N

   (d) 50 N

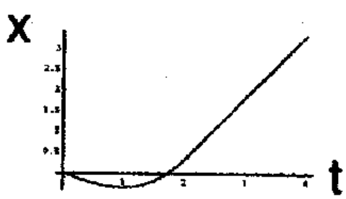
   (e) 60 N



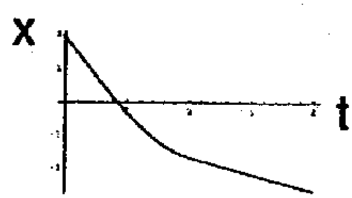
3. An object starts with an initial velocity in the negative direction and then moves under the influence of a net force that is in the positive direction and whose magnitude as a function of time is depicted to the right. Which of the graphs below could depict the position of the object as a function of time?



I



II



III

   (a) only I or II or III

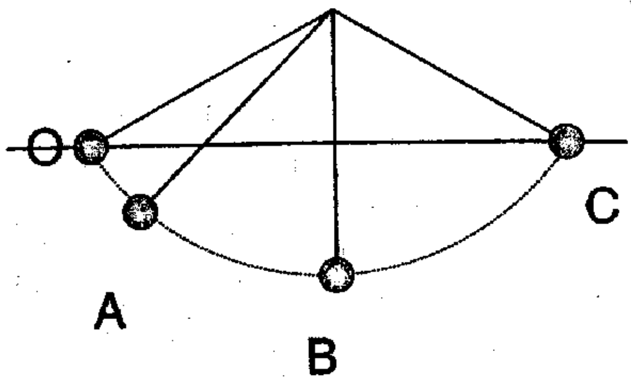
   (b) I and II

   (c) II and III

   (d) I and III

   (e) I, II and III

4. Consider the figure depicting the path of a pendulum. The pendulum is released from rest at point O. Which of the following statements are true?
I. At point A, the velocity and acceleration are perpendicular.
II. At point B, the normal acceleration has its greatest magnitude.
III. At point C, the tangential acceleration has a greater magnitude than at point A and point B.



(a) only I or II or III
(b) I and II
(c) II and III
(d) I and III
(e) I, II and III

5. A person is pushing a block on a level floor with friction. The person and block are *accelerating* together. Which of the following statements accurately characterize this situation?

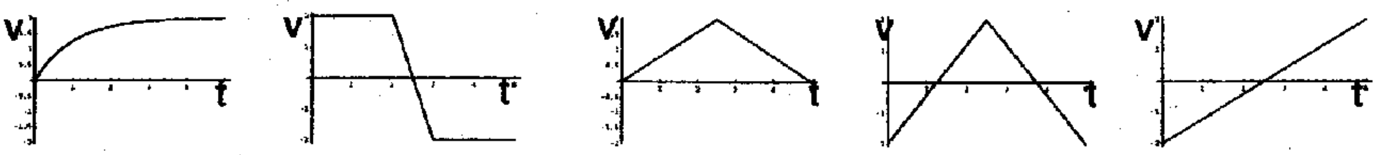
I. The force that the person exerts on the block must be greater than the force of the block on the person for acceleration to occur.

II. The force that the person exerts on the block is equal to the friction opposing the motion of the block.

III. The force that the person exerts on the ground is equal to the force the person exerts on the block.

(a) only I or II or III
(b) I and II
(c) II and III
(d) I and III
(e) None are true.

For questions 6 and 7: Consider the following graphs of velocity versus time. (The scales of the axes are the same in each graph.)

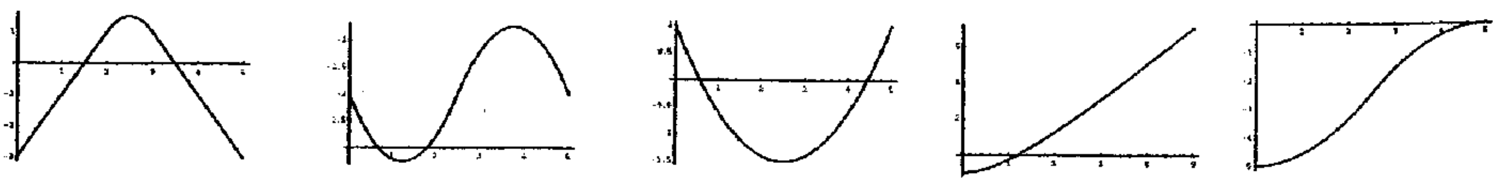


1 2 3 4 5

6. Which particle undergoes the largest displacement during the time interval shown?

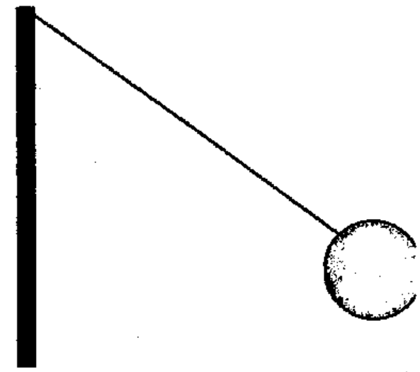
(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

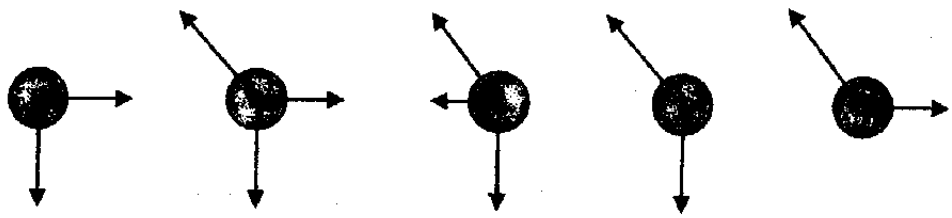
7. Which of the position-time graphs below matches with velocity graph 4?



A B C D E

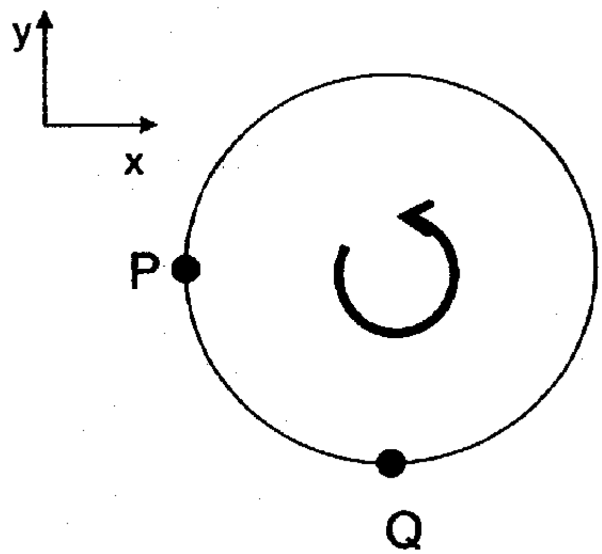
8. A rock is tied by a string to a pole. The rock is given a kick and it swings around the pole. At the moment depicted, the rock is in the plane of the page and the motion of the rock is out of the page. Which of the free-body-diagrams below accurately represents this physical situation?





A. B. C. D. E.

For questions 9-10: A merry-go-round rotates at a constant rate in the counter-clockwise direction. Use the coordinate axes labelled next to the figure. The outer edge is a distance R from the center and the merry-go-round makes a complete revolution in a time T. A carnival worker sitting on the edge rides from point P to point Q.



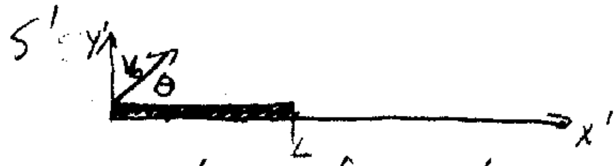
9. What is the average velocity from point P to point Q?

   (a) $\frac{4R}{T}(\hat{\imath} - \hat{\jmath})$
   (b) $-\frac{4\sqrt{2}R}{T}(\hat{\imath} + \hat{\jmath})$
   (c) $\frac{4\sqrt{2}R}{T}(\hat{\imath} - \hat{\jmath})$
   (d) $-\frac{\pi R}{T}(\hat{\imath} + \hat{\jmath})$
   (e) $\frac{\pi R}{T}(-\hat{\imath} + \hat{\jmath})$

10. What is the average acceleration from point P to point Q?

   (a) $\frac{4\pi^2 R}{T^2}(\hat{\imath} + \hat{\jmath})$
   (b) $\frac{4\pi^2 R}{T^2}(-\hat{\imath} + \hat{\jmath})$
   (c) $\frac{8\pi R}{T^2}(\hat{\imath} + \hat{\jmath})$
   (d) $-\frac{8\pi R}{T^2}(\hat{\imath} + \hat{\jmath})$
   (e) $\frac{4\pi R}{T^2}(\hat{\imath} - \hat{\jmath})$

1. a.



reference frame of train

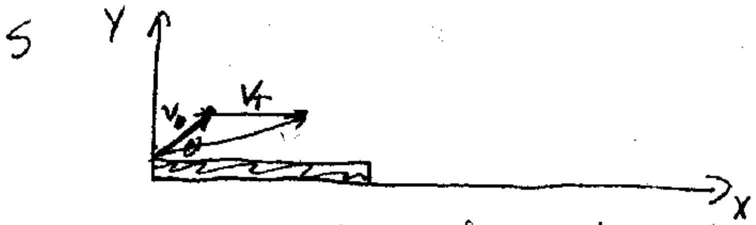
$V_b = 55\,m/s$

$V_t = 30\,m/s$

$\theta = 50^\circ$

$g = 9.8\,m/s$

$L = 0.4\,m$



reference frame of earth

For time, horizontal component irrelevant

Use $y'(t) = V_b \sin\theta\, t - \frac{1}{2} g t^2$

or $y(t) = V_b \sin\theta\, t - \frac{1}{2} g t^2$

or $V_y'(t) = V_b \sin\theta - g t$

or $V_y(t) = V_b \sin\theta - g t$

call landing time t_ℓ

then either: $y'(t_\ell) = y(t_\ell) = 0 = V_b \sin\theta\, t_\ell - \frac{1}{2} g t_\ell^2$

solve $\Rightarrow t_\ell = \frac{2 V_b \sin\theta}{g} = \boxed{8.6\,s}$

or: $V_y'\left(\frac{t_\ell}{2}\right) = V_y\left(\frac{t_\ell}{2}\right) = 0 = V_b \sin\theta - g \frac{t_\ell}{2}$

$\Rightarrow t_\ell = \frac{2 V_b \sin\theta}{g} = \boxed{8.6\,s}$

or: $V_y'(t_\ell) = V_y(t_\ell) = -V_b \sin\theta = V_b \sin\theta - g t_\ell$

$\Rightarrow t_\ell = \frac{2 V_b \sin\theta}{g} = \boxed{8.6\,s}$

1. b. one possible starting point

$$\vec{V}_{BE} = \vec{V}_{BT} + \vec{V}_{TE}$$

$$V_{BT,x} = V_b \cos\theta \qquad V_{BT,y} = V_b \sin\theta$$

$$V_{TE,x} = V_t \qquad V_{TE,y} = 0$$

$$\Rightarrow \quad V_{BE,x} = V_b\cos\theta + V_t \qquad V_{BE,y} = V_b \sin\theta$$

$$\Rightarrow \quad X_{BE}(t) = (V_b\cos\theta + V_t)t$$

$$\Rightarrow \quad X_{BE}(t_\ell) = (V_b\cos\theta + V_t)t_\ell = (V_b\cos\theta + V_t)\frac{2V_b\sin\theta}{g}$$

$$\boxed{= 560\,m}$$

or could have use

$$R' = \frac{V_b^2 \sin 2\theta}{g} = 300m$$

range in train frame

$$R = R' + V_t t_\ell = \frac{V_b^2 \sin 2\theta}{g} + \frac{2V_b V_t \sin\theta}{g}$$

$$= 560m$$

$$X_{Front}(t) = L + V_T t$$

$$X_{Front}(t_\ell) = L + V_t t_\ell = L + \frac{2V_b V_t \sin\theta}{g}$$

$$\underline{= 660m}$$

$$X_{back}(t) = V_T t$$

$$\underline{= 260m}$$

1.c. we want train to stop when

$$X_{Front}(t_\ell) = X_{BE}(t_\ell) \rightarrow \text{find } a_T$$

$$X_{Front}(t_\ell) = L + V_T t_\ell - \frac{1}{2} a_T t_\ell^2$$

$$X_{BE}(t_\ell) = (V_b \cos\theta + V_T) t_\ell$$

setting equal, cancelling $V_T t_\ell$

$$L - \frac{1}{2} a_T t_\ell^2 = V_b \cos\theta \, t_\ell$$

$$\Rightarrow \quad \frac{1}{2} a_T t_\ell^2 = L - V_b \cos\theta \, t_\ell$$

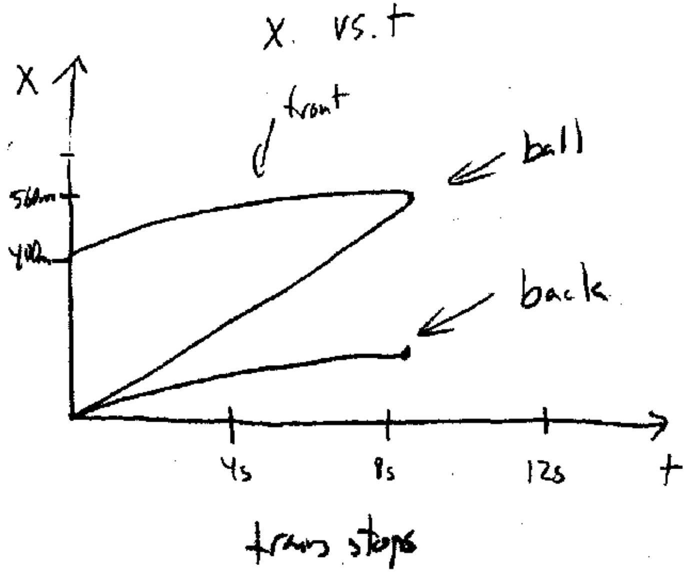
$$a_T = \frac{2(L - V_b \cos\theta \, t_\ell)}{t_\ell^2}$$

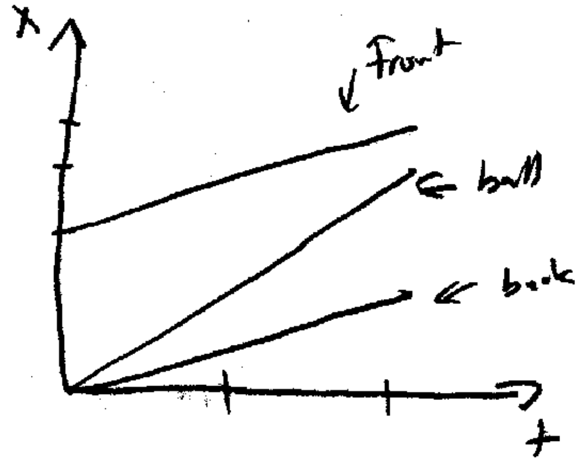
$$= \frac{2\left(L - \frac{V_b^2 \sin 2\theta}{g}\right)}{\frac{4 V_b^2 \sin^2\theta}{g^2}}$$

$$a_T = 2.6 \text{ m/s}^2$$

in the $-x$ direction

1.d.





1.a. Required: $t_e = 8.6$ s (must have units) +8

(8) Partial Credit

picture	+1
parametric equation	+1
correctly	+1
setting equal to critical value	+1
w/ correctly	+1
solving	+1
correctly	+1
with units + sig. Figs	+1
	8

1.b. (12) Required

$x_{BE}(t_e) = 560 m$ +6

$x_{Front}(t_e) = 660 m$ +3

$x_{back}(t_e) = 260 m$ +3

Partial Credit

finding v_{BE}	+2	> 6
parametric eq $x_{BE}(t)$	+2	
solving correct	+1	
units	+1	
p. eq. for $x_{Front}(t_e)$	+2	> 3
solving w/ units	+1	
p. eq. for $x_{back}(t_e)$	+2	> 3
solving w/ units	+1	

1.c. (7) Required $a_T = 2.6 m/s^2$ in $-x$ direction

number	+4
units	+1
direction	+1

Partial Credit

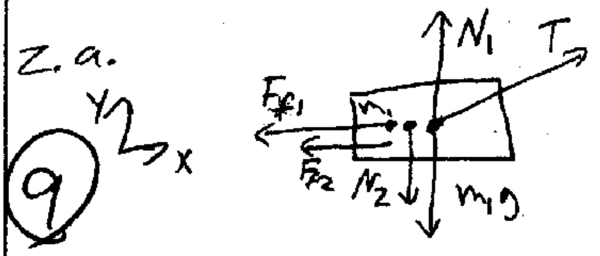
parametric eq. for $x_{Front}(t_e)$	+2	> 7
" " for $x_{BE}(t_e)$	+1	
setting them equal	+1	
solving	+1	
units	+1	
direction	+1	

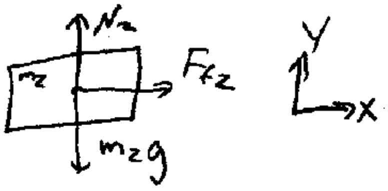
1.d. (8) Required v_T constant

straight ball, higher slope	+1	+8
straight train	+1	
spacing	+1	
labels	+1	
straight ball	+1	
concave down train	+1	
spacing	+1	
labels	+1	

2. a.

(9)





N_1 - normal force of floor
$m_1 g$ - weight of m_1
T - tension
N_2 - normal force of m_2
F_{f1} - friction of m_1 on floor
F_{f2} - friction of m_1 on m_2

N_2 - normal force of m
$m_2 g$ - weight of m_2
F_{f2} - friction of m_2 on

2b.

(8)

$\Sigma F_{1y} = m_1 a_{1y}$

$N_1 + T\sin\theta - N_2 - m_1 g = m_1 a_{1y}$ $\quad a_{1y} = 0$

$\Sigma F_{1x} = m_1 a_{1x}$

$T\cos\theta - F_{f1} - F_{f2} = m_1 a_{1x}$

$\Sigma F_{2y} = m_2 a_{2y}$

$N_2 - m_2 g = m_2 a_{2y}$ $\quad a_{2y} = 0$

$\Sigma F_{2x} = m_2 a_{2x}$

$F_{f2} = m_2 a_{2x}$

2.c. (7)

then $a_{2x} = a_{1x} = 0$

so $F_{f2} = 0$, $N_2 = m_2 g$

and $F_{f1} = \mu_k N_1$

and $T\cos\theta = F_{f1}$

$$N_1 + T\sin\theta - m_2 g - m_1 g = 0$$

$$\Rightarrow N_1 = (m_1 + m_2)g - T\sin\theta$$

$$\Rightarrow T\cos\theta = \mu_k (m_1 + m_2) g - \mu_k T\sin\theta$$

$$\Rightarrow T(\cos\theta + \mu_k \sin\theta) = \mu_k (m_1 + m_2) g$$

$$\boxed{T = \frac{\mu_k (m_1 + m_2) g}{\cos\theta + \mu_k \sin\theta}}$$

2.d. (5)

$a_{2x} = a_{1x} = a$

$F_{f2} \leq \mu_s N_2$

$$\Rightarrow \mu_s m_2 g \geq m_2 a$$

$$\Rightarrow a \leq \mu_s g$$

so $\boxed{a_{max} = \mu_s g}$

2.e. (6)

so. $N_2 = m_2 g$ still $\quad a_{1x} = a_{2x} = a_{max} = \mu_s g$

$$N_1 + T\sin\theta - m_2 g - m_1 g = 0$$

$$\Rightarrow N_1 = (m_1+m_2)g - T\sin\theta$$

and $T\cos\theta - \mu_k N_1 - \mu_s m_2 g = m_1 \mu_s g$.

$$\Rightarrow T\cos\theta - \mu_k (m_1+m_2)g + \mu_k T\sin\theta = \mu_s (m_1+m_2)g$$

$$\Rightarrow T(\cos\theta + \mu_k \sin\theta) = \mu_s (m_1+m_2)g + \mu_k (m_1+m_2)g$$

$$\boxed{T = \frac{(m_1+m_2)(\mu_s+\mu_k)g}{\cos\theta + \mu_k \sin\theta}}$$

2.a. (9) Required:

each force + description +1 (x 9)

→ subtract 1 for each extra force

2.b. (8) Required:

each component of N's Law +2

2.c. Required: $T = \frac{\mu_k (m_1+m_2) g}{\cos\theta + \mu_k \sin\theta}$

(7)

Partial

$a_{2x} = a_{1x} = 0$	+1
$F_{f2} = 0$	+1
$F_{f1} = \mu_k N_1$	+1
$N_2 = m_2 g$	+1
$T\cos\theta = F_{f1}$	+1
expression for N_1	+1
solving for T	+1

2.d. Required $a_{max} = \mu_s g$ +5

(5) Partial

$F_{f2} = m_2 a_{2x}$	+1
$F_{f2} \leq \mu_s N_2$	+1
$N_2 = m_2 g$	+1
solving for a_{2x}	+1
finding a_{max}	+1

2.e. Required $T = \frac{(m_1+m_2)(\mu_s+\mu_k)g}{\cos\theta + \mu_k \sin\theta}$

(6)

$a_{1x} = a_{2x} = a_{max}$	+1
using: $N_1 + T\sin\theta - N_2 - m_1 g = 0$	+1
using $T\cos\theta - F_{f1} - F_{f2} = m_1 a_{max}$	+1
subbing in things	+1
solving for T	+2

Last Name: KEY First Name: ____________

Physics 101 Fall 2002: Test 1—Multiple-Choice Answers

	A	B	C	D	E
1		X			
2			X		
3			X		
4			X		
5					X
6	X				
7		X			
8				X	
9	X				
10			X		