Last Name: ______________________ First Name: ______________________

Physics 101 Fall 2001: Final—Free Response and Instructions

2:00-5:00 PM, 12 December, 2001

- **Print your LAST and FIRST name on the front of your blue book, on this question sheet and the multiple-choice answer sheet.**
- **TIME ALLOWED 180 MINUTES**
- **The test consists of four free-response questions and fifteen multiple-choice questions.**
- **The test is graded on a scale of 200 points; the first free-response question accounts for 25 points, the second for 35 points, the third and fourth for 40 points and the multiple-choice questions account for 60 points.**
- **Answer the four free-response questions in your blue book. Answer the multiple-choice questions by marking a dark X or Roman numeral in the appropriate column and row in the table on the multiple-choice and matching answer sheet.**
- **Consult no books or notes of any kind. You may use a hand-held calculator in non-graphing, non-programmed mode.**
- **Do NOT take test materials outside of the class at any time. Return this question sheet along with your blue book and multiple choice answer sheet.**
- **Write and sign the Pledge on the front of your blue book.**

Show your work for the free-response problems, including neat and clearly labelled figures, in your blue book. Answers without explanation (even correct answers) may not receive full credit.

Throughout the test, when necessary use $g = 9.8$ m/s^2.

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1. **[25 pts] A stock car pit crew needs to select the right tires for their car to compete at a new super speedway. With the correct tires, their car needs not slow down when rounding the curves. The speedway is a 3.0 km long oval with corners having radii of curvature of 250 m. During the race, cars travel at 262 km/hr. The banking angle in the corners is measured to be 22° above the horizontal. Assume no air resistance and that the tires roll without slipping on the track.**

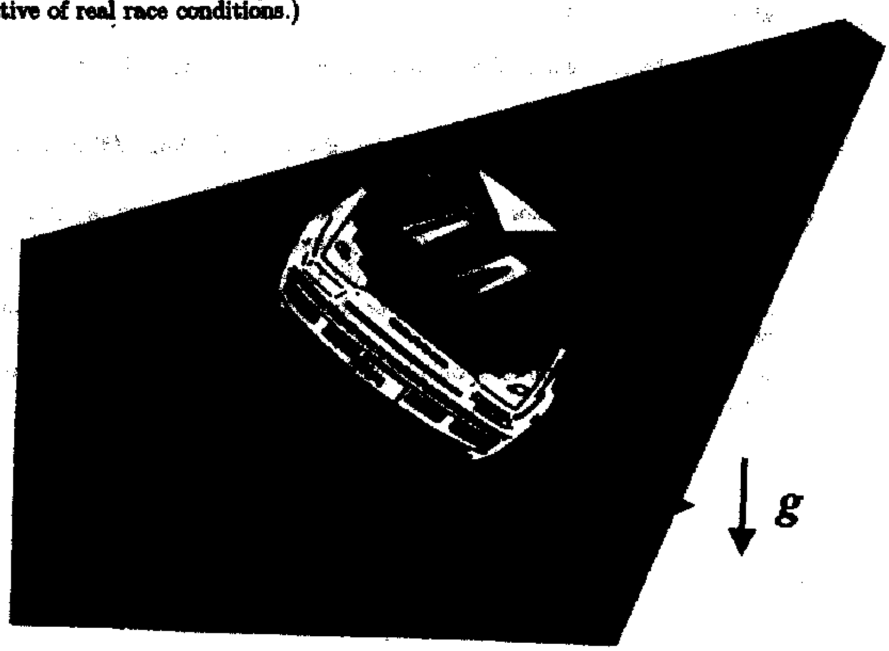
   (a) **Draw a free-body diagram for a race car as it rounds the corner at racing speed. Briefly describe all forces present in your free-body diagram.**

   (b) **Use your free body diagram to write down equations for Newton's 2nd Law in the vertical and horizontal directions.**

   (c) **Determine the minimum coefficient of friction between the roadway and the tires that allows the race cars to negotiate the curve at race speed.**

   (d) **Calculate the amount of work done by the frictional force in one lap.**

   (Note: real race cars take advantage of aerodynamic forces to increase the normal force, so your answers are not reflective of real race conditions.)



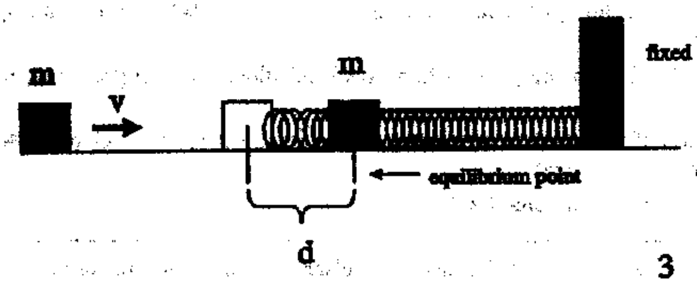
2. [35 pts] A block of mass m resting on a horizontal friction-free surface is attached by a spring of spring constant k to a fixed object to its right as shown in the figure. An external force does work to pull the block a distance d to the left of its equilibrium position and then releases it at time $t = 0$.

(a) In a frame of reference in which the origin taken to be the equilibrium point, write the equations of motion for position $x(t)$, velocity $v(t)$ and acceleration $a(t)$ of the block in terms of m, k and d.

(b) Now suppose that when the block is at its furthest extension to the left it is struck by another block also of mass m moving with speed v_0 to the right. If the blocks stick together, determine the amplitude and period of oscillation after the collision in terms of m, k, v_0 and d.

(c) Use the values $v_0 = 11$ m/s, $d = 0.8$ m, $m = 9$ kg and $k = 750$ N/m. Determine the energy lost in the collision above. How much energy would be lost if instead the blocks had collided when the block attached to the spring was passing through the equilibrium position moving to the right?

2



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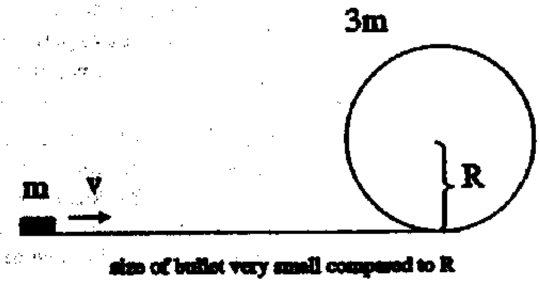


3. [40 pts] A disk of mass $M = 3m$ and radius R lies flat on a frictionless horizontal surface. A bullet of mass m is fired with speed v_0 (see figure) so that it strikes the rim of the disk tangentially and becomes embedded. The moment of inertia of a disk about a perpendicular axis through its center of mass is $I = MR^2/2$.

First consider the case where the disk is anchored by a pin through its center of mass so that it can rotate freely about its center but it cannot be displaced.

(a) What is the final angular velocity ω about the pin of the bullet/disk system in terms of some or all of m, R and v_0?

Now consider the case where the pin is removed and the bullet/disk system can move freely after the collision.

(b) What is the final velocity of the center of mass of the bullet/disk system in terms of some or all of m, R and v_0?

(c) Where is the new center of mass of the bullet/disk system in terms of m and R? Show that the moment of inertia of the bullet/disk system about an axis passing through this center of mass and perpendicular to the surface is $I = (9/4)mR^2$.

(d) Determine the final angular velocity ω about the center of mass of the bullet/disk system in terms of some or all of m, R, v?

(e) In which case does the disk/bullet system end up with more kinetic energy? Give a brief justification.

4. **[40 pts]** A block of mass m_1 and uniform solid are released from rest and accelerate down a slope that makes an angle θ with the horizontal (see figure 4a). The coefficient of kinetic friction between the block and the slope is μ_k. The moment of inertia of a solid cylinder about its central axis is $MR^2/2$.

   (a) Find the acceleration of the block. Express your answer in terms of some or all of m_1, g, θ and μ_k.

   (b) Assuming the cylinder rolls without slipping, find the acceleration of the center of mass of the cylinder. Express your answer in terms of some or all of m_2, g, θ and R.

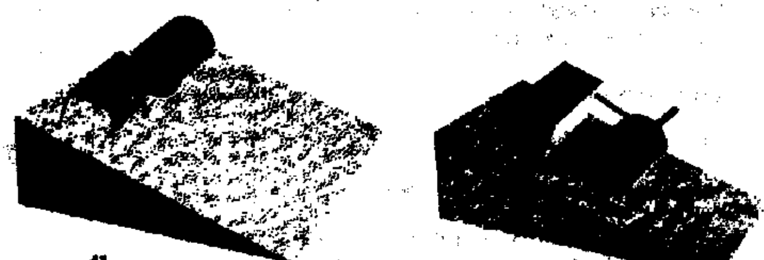
   Now the block and cylinder are attached by light strings (as shown in the figure 4b) that are attached to the cylinder's axle with frictionless loops so that the cylinder can roll without a torque about the axle from the strings. The combined system is released from rest.

   (c) Find the acceleration of the system, assuming that the strings remain taut and that the cylinder rolls without slipping. Express your answer in terms of some or all of m_1, μ_k, m_2, g, θ and R.

   (d) Determine the minimal coefficient of static friction between the cylinder and slope necessary for the cylinder to roll without slipping for the following values: $m_1 = 4$ kg, $\mu_k = 0.3$, $m_2 = 7$ kg, $g = 9.8$ m/s, $\theta = 20°$ and $R = .3$ m.

   (e) Is the assumption that the strings remain taut justifiable? Give a brief explanation.

4



4b   4b

Multiple-Choice

1. A small block of mass m is released from rest a small way from the bottom of a frictionless semicircular bowl of radius R. Its original height above the bottom of the bowl is h; the original angle from the vertical is θ. Approximately how long does it take for the small mass to return to the same height on the same side?

   (a) $2\sqrt{\frac{2h}{g}}$

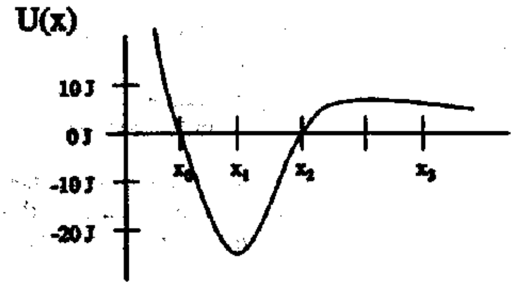
   (b) $2\sqrt{\frac{\sin\theta g}{R}}$

   (c) $2\pi\sqrt{\frac{R}{g}}$

   (d) $2\sqrt{\frac{\theta R}{\sin\theta g}}$

   (e) $2\sqrt{\frac{2\pi R}{g}}$

2. A conservative force has the potential energy function $U(x)$ as shown by the graph. A particle moving in one dimension under the influence of this force has a kinetic energy of 10 J at x_1. Which of the following is a correct statement about the subsequent motion of the particle?



   (a) It oscillates with maximum position x_2 and minimum position x_0.

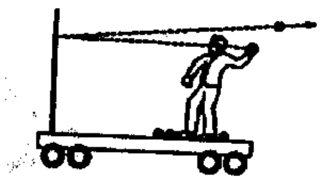
   (b) It moves to right of x_3 and never returns.

   (c) It moves to the left of x_0 and never returns.

   (d) It comes to rest at either x_0 or x_2 depending on the initial direction of its velocity.

   (e) It cannot reach either x_0 or x_2.

3. Suppose you are on a cart, initially at rest on a track with very little friction. You throw a ball at a partition that is rigidly mounted on the cart. If the ball bounces straight back as shown in the figure, is the cart put in motion?



   (a) Yes, it moves to the right.

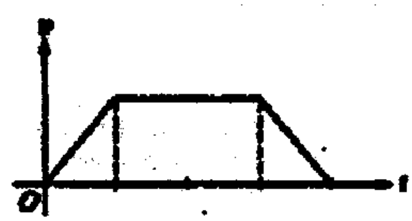
   (b) Yes, it moves to the left.

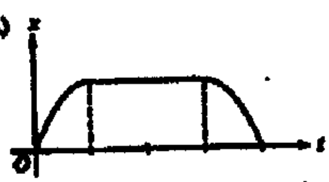
   (c) No, it remains in place at all times.

   (d) The cart ends up at rest, but it moves to the right during the interval between the throw and the bounce.

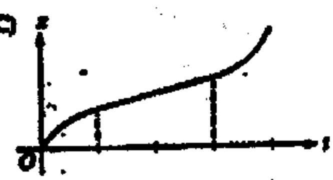
   (e) The cart ends up at rest, but it moves to the left during the interval between the throw and the bounce.

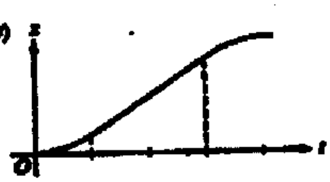
4. The graph to the left shows the velocity v as a function of time t for an object moving in a straight line. Which of the following graphs shows the displacement x as a function of time t for the same time interval?



(A)


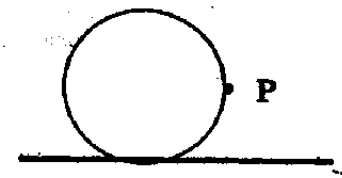
(B)


(C)


(D)


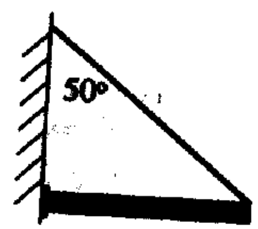
(E)


5. A ball rolls without slipping along level ground. If the center of mass of the ball has a speed of 3 m/s with respect to the ground, what is the instantaneous speed of the frontmost point P on the ball?



(a) zero
(b) 3/2 m/s
(c) 3 m/s
(d) $3\sqrt{2}$ m/s
(e) 6 m/s

6. A uniform rod of mass $m = 50$ kg and length $L = 2.6$ m is supported by a hinge at the wall and a cord attached to its end. The rod is level and the cord makes an angle of $\phi = 50°$ with the vertical. What is the tension in the cord?



(a) zero
(b) about 245 N
(c) about 320 N
(d) about 380 N
(e) about 490 N

7. A rock is dropped from rest off a cliff and it falls the first half of the distance to the ground in t_1 seconds. If it falls the second half of the distance in t_2 seconds, what is the value of t_2/t_1? (Ignore air resistance.)

(a) Indeterminate. You need to know the height of the cliff.
(b) 1/2
(c) $1/\sqrt{2}$
(d) $1 - (1/\sqrt{2})$
(e) $\sqrt{2} - 1$

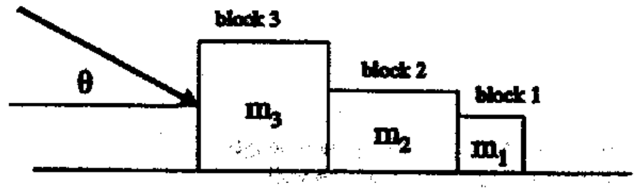
8. Assuming circular orbits for Earth and Mars, which one of the following quantities is the same for both planets with respect to an axis passing through the center of the orbit perpendicular to the plane of orbit?

(a) angular velocity
(b) tangential speed
(c) angular acceleration
(d) force of gravity exerted by planet on sun
(e) force of gravity exerted by sun on planet

9. A food bomb of mass 6 kg initially at rest explodes into three packets of tasty meals. The breakfast part has mass 2 kg and velocity $v_{breakfast} = (2\text{ m/s})\,\hat{\imath} - (1\text{ m/s})\,\hat{\jmath} + (4\text{ m/s})\,\hat{k}$ and the lunch part has mass 1 kg and velocity $v_{lunch} = (3\text{m/s})\hat{\jmath} + (-2\text{ m/s})\,\hat{k}$. What is the $\hat{k}$-component of v_{dinner}

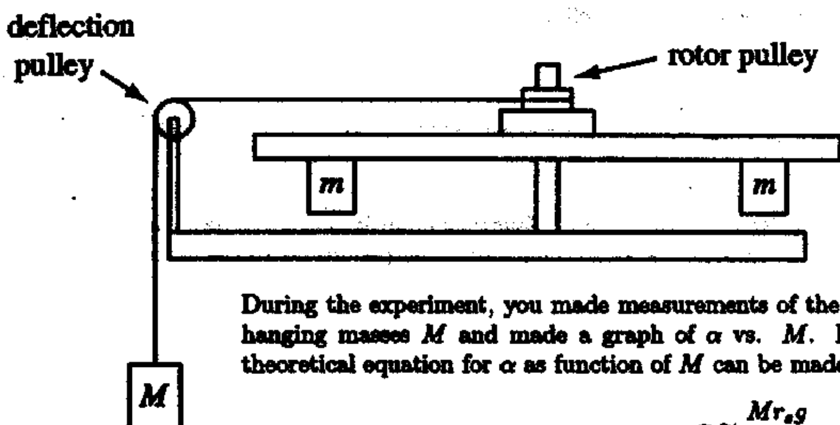
(a) -6 m/s
(b) -3 m/s
(c) -2 m/s
(d) 2 m/s
(e) 6 m/s

10. Consider the blocks below, initially at rest on a frictionless surface. The blocks have the masses m_1, $m_2 = 3m_1$ and $m_3 = 5m_1$. If the force of magnitude F acting at the shown angle θ pushes to blocks moving them a distance d, how much work is done by block 2 on block 1?



(a) none
(b) $\frac{Fd\cos\theta}{9}$
(c) $\frac{3Fd\sin\theta}{5}$
(d) $\frac{8Fd}{9}$
(e) $\frac{8Fd\cos\theta}{9}$

11. Consider the experimental apparatus that you used in the laboratory part of this class, experiment 5, depicted below.



During the experiment, you made measurements of the angular acceleration α of the bar for various hanging masses M and made a graph of α vs. M. By making one or more approximations, the theoretical equation for α as function of M can be made linear:

$$\alpha \approx \frac{Mr_s g}{I} - \frac{\tau_f}{I},$$

where I is the moment of inertia of the rotating system, τ_f is the resistive torque and r_s is the radius of the rotor pulley. What is (are) the approximation(s) required to justify the linear relationship?

I. The tension in the string is equal to the weight of the hanging mass.

II. The bar is massless.

III. The resistive torque is independent of angular velocity.

(a) I
(b) II
(c) III
(d) I and II
(e) I and III

12. A small metal sphere of mass m is launched horizontally with speed v from a height h above a smooth, level surface. The ball strikes the ground and rebounds to a height $(2/3)h$. What impulse is delivered to the sphere by the ground during the collision?

(a) none

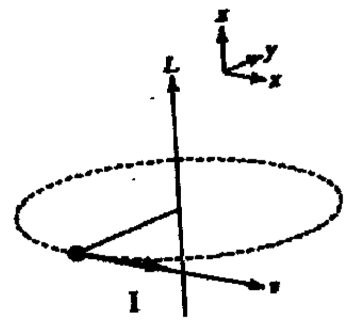
(b) $m(\sqrt{2gh} - \sqrt{\frac{4}{3}gh})$

(c) $m(\sqrt{2gh} + \sqrt{\frac{4}{3}gh})$

(d) $m(\sqrt{v^2 + \frac{4}{3}gh} + \sqrt{v^2 + 2gh})$

(e) $2mv$

13. In deep space, a person spins a tennis ball on a string in a circle in the x-y place (as shown in the figure). At the point indicated, the ball is given an impulse in the forward direction. This causes a change in angular momentum ΔL in the



(a) +x direction

(b) +y direction

(c) -y direction

(d) +z direction

(e) -z direction

14. At a given instant, an observer stationary on the ground sees a package falling with speed v_1 at an angle to the vertical. A pilot flying at a constant horizontal speed relative to the ground sees the package falling vertically with a speed v_2 at the same instant. What is the speed of the pilot relative to the ground?

(a) $\sqrt{v_1^2 + v_2^2}$

(b) $\sqrt{v_1^2 - v_2^2}$

(c) $v_1 - v_2$

(d) $v_1 + v_2$

(e) $v_2 - v_1$

15. A truck breaks down out on the road and receives a push back into town by a compact car. While the car, still pushing the truck, is speeding up to cruising speed,

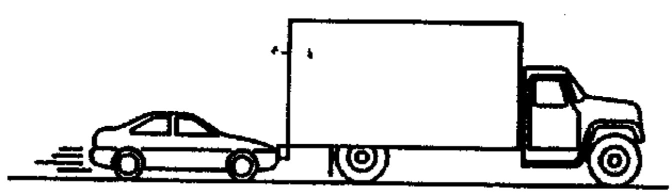
(a) the amount of force with which the car pushes on the truck is equal to that with which the truck pushes back on the car.

(b) the amount of force with which the car pushes on the truck is smaller than that with which the truck pushes back on the car.

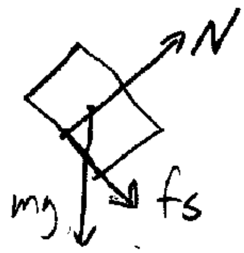
(c) the amount of force with which the car pushes on the truck is greater than that with which the truck pushes back on the car.

(d) the force of the car on the truck cancels the force of the truck on the car and so there is zero net force.

(e) the car's engine is running and therefore exerts a force on the truck but the truck's engine is not running and therefore it cannot exert a force on the car.



1.a

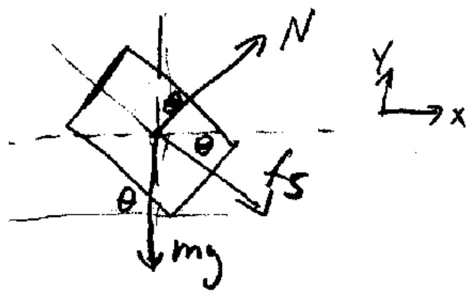


N – normal force

mg – gravitational force

f_s – static friction

b.



x: $N\sin\theta + f_s\cos\theta = ma_x$

$$a_x = \frac{v^2}{R}$$

y: $N\cos\theta - mg - f_s\sin\theta = ma_y$

$$a_y = 0$$

c. so $N\sin\theta + f_s\cos\theta = \frac{mv^2}{R}$ (1)

$N\cos\theta - mg - f_s\sin\theta = 0$ (2)

for minimum μ_s, $f_s = \mu_s N$

(1) $\Rightarrow$ $N(\sin\theta + \mu_s\cos\theta) = \frac{mv^2}{R}$

(2) $\Rightarrow$ $N(\cos\theta - \mu_s\sin\theta) = mg$

so $N = \frac{mg}{\cos\theta - \mu_s\sin\theta}$

substituting

$\Rightarrow$ $mg(\sin\theta + \mu_s\cos\theta) = \frac{mv^2}{R}(\cos\theta - \mu_s\sin\theta)$

collecting terms

$$mg\cos\theta\,\mu_s + \frac{mv^2}{R}\sin\theta\,\mu_s = \frac{mv^2}{R}\cos\theta - mg\sin\theta$$

$$\Rightarrow \mu_s = \frac{\frac{mv^2}{R}\cos\theta - mg\sin\theta}{mg\cos\theta + \frac{mv^2}{R}\sin\theta} = \frac{v^2\cos\theta - gR\sin\theta}{gR\cos\theta + v^2\sin\theta} =$$

c. cont

$$v = 262\,km/hr = 72.8\,m/s$$

$$\Rightarrow \boxed{\mu_s = .938}$$

d.

$$W_{net} = \Delta KE$$

$$\Delta KE = 0$$

so $W_{net} = 0$

$$W_{net} = W_g + W_N + W_{f_s}$$

$W_g = 0$ (no change in height)

$W_N = 0$ (no displacem in normal direction)

so $\boxed{W_{f_s} = 0}$ (alternatively, no displacement in direction of f_s)

2. a. $x(t) = A\cos(\omega t + \phi)$

$A = d \quad \omega = \sqrt{\frac{k}{m}} \quad \phi = 0$

so

$$\underline{x(t) = d\cos\left(\sqrt{\tfrac{k}{m}}\,t\right)}$$

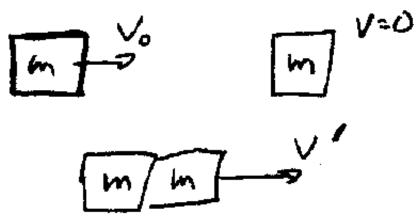
$$\underline{v(t) = \frac{dx}{dt} = -d\sqrt{\tfrac{k}{m}}\sin\left(\sqrt{\tfrac{k}{m}}\,t\right)}$$

$$\underline{a = \frac{dv}{dt} = -\frac{dk}{m}\cos\left(\sqrt{\tfrac{k}{m}}\,t\right)}$$

b. Inelastic collision!

befor

after



so $P_i = P_f$

$$mV_0 = 2mV'$$

$$V' = \frac{V_0}{2}$$

One way to solve problem is energy

After collision $KE_A = \frac{1}{2}(2m)V'^2 = \frac{1}{4}mV_0^2$

$$PE_A = \frac{1}{2}kd^2$$

At furthest displacement (new amplitude A')

$$KE_B = 0 \qquad PE_B = \frac{1}{2}kA'^2$$

so $\frac{1}{2}kA'^2 = \frac{1}{4}mV_0^2 + \frac{1}{2}kd^2$

$$A'^2 = \frac{1}{2}\frac{m}{k}V_0^2 + d^2$$

$$\underline{A' = \sqrt{\frac{1}{2}\frac{m}{k}V_0^2 + d^2}}$$

$$\underline{\omega' = \sqrt{\frac{k}{2m}}}$$

c. all energy lost in inelastic collision

$$KE_i = \frac{1}{2} m v_0^2 \qquad KE_f = \frac{1}{2}(2m)v'^2 = \frac{1}{4} m v_0^2$$

$$\underline{\Delta KE = -\frac{1}{4} m v_0^2 = -270\,J}$$

if had collided at $x=0$

before

after

$$p_i = m v_0 + m d\sqrt{\frac{k}{m}}$$

$$p_f = 2m v'$$

$$\Rightarrow \quad v' = \frac{v_0}{2} + \frac{d}{2}\sqrt{\frac{k}{m}}$$

$$KE_i = \frac{1}{2} m v_0^2 + \frac{1}{2} m \left(d\sqrt{\frac{k}{m}}\right)^2 = \frac{1}{2} m v_0^2 + \frac{1}{2} k d^2$$

$$KE_f = \frac{1}{2}(2m)\left(\frac{v_0}{2} + \frac{d}{2}\sqrt{\frac{k}{m}}\right)^2 = m\left(\frac{v_0^2}{4} + \frac{v_0 d}{2}\sqrt{\frac{k}{m}} + \frac{d^2 k}{4m}\right)$$

$$KE_f - KE_i = \frac{v_0 d}{2}\sqrt{km} - \frac{1}{4} m v_0^2 - \frac{1}{4} k d^2$$

$$= 360\,J - 270\,J - 120\,J$$

$$\underline{= -30\,J}$$

3. a. Use cons. ang. mom

$L_i = mvR$

$L_f = I_{tot}\,\omega = (I_{disk} + I_{bullet})\,\omega$

$= \left(\frac{1}{2}(3m)R^2 + mR^2\right)\omega$

$= \frac{5}{2} mR^2\omega$

$\Rightarrow \underline{\omega = \frac{2}{5}\frac{v}{R}}$

b. cons. mom

$p_i = mv \qquad p_f = (m+3m)v'$

$\Rightarrow \underline{v' = \frac{v}{4}}$

c.

$r_{cm} = \frac{3m\cdot\phi + mR}{3m+m} = \frac{R}{4}$

$I_{CM} = I_{disk,cm} + I_{bull,CM}$

$I_{disk,cm} = \frac{1}{2}(3m)R^2 + 3m\left(\frac{R}{4}\right)^2 \quad \leftarrow$ // axis thm

$= \frac{3}{2}mR^2 + \frac{3}{16}mR^2$

$= \frac{27}{16}mR^2$

$I_{bull,cm} = m\left(\frac{3R}{4}\right)^2 = \frac{9}{16}mR^2$

$I_{cm} = \left(\frac{27}{16} + \frac{9}{16}\right)mR^2 = \boxed{\frac{9}{4}mR^2}$

(Alternative

$I_{CM} + M_{tot}R_{cm}^2 = I_{tot}$

$\Rightarrow I_c = \frac{5}{2}mR^2 - 4m\left(\frac{R}{4}\right)^2 = \underline{\frac{9}{4}mR^2}$

about new CM

d. $L_i = mvR_{cm} = \frac{3}{4} mvR$

$L_f = I_{cm}\,\omega = \frac{9}{4} mR^2\omega$

$\Rightarrow \quad \omega = \frac{1}{3}\frac{v}{R}$

e. In the second case.

Reasons:

1) In the second collision, the bullet will travel further while embedding, so more work will be done.

2) In the second case, there will be no work done to decrease energy by external force at pivot.

3) Explicit calc

$KE_1 = \frac{1}{2} I_{tot}\,\omega_1^2 = \frac{1}{2}\left(\frac{5}{2}mR^2\right)\left(\frac{2}{5}\frac{v}{R}\right)^2$

$= \frac{1}{5} mv^2$

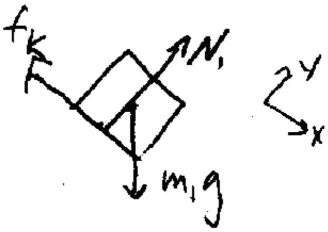
$KE_2 = \frac{1}{2} I_{cm}\,\omega_2^2 + \frac{1}{2} m_{tot} v'^2$

$= \frac{1}{2}\left(\frac{9}{4}mR^2\right)\left(\frac{1}{3}\frac{v}{R}\right)^2 + \frac{1}{2}\,4m\left(\frac{v}{4}\right)^2$

$= \frac{1}{8}mv^2 + \frac{1}{8}mv^2 = \frac{1}{4}mv^2$

so $KE_2 > KE_1$

4. a.



y: $N_1 - mg\cos\theta = 0$

x: $mg\sin\theta - f_k = ma$

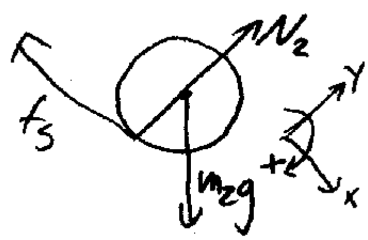
$f_k = \mu_k N_1$

so $mg\sin\theta - \mu_k N_1 = ma$

$\Rightarrow \quad mg(\sin\theta - \mu_k\cos\theta) = ma$

$\Rightarrow \quad \underline{a = g(\sin\theta - \mu_k\cos\theta)}$

b.



y: $N_2 - m_2 g\cos\theta = 0$

x: $m_2 g\sin\theta - f_s = m_2 a$

τ: $f_s R = I\alpha$

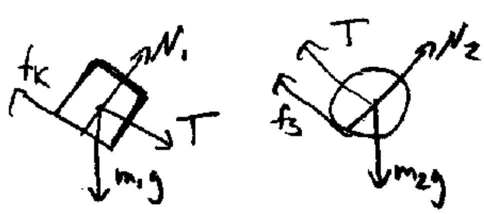
$\alpha = \frac{a}{R}$

$\Rightarrow \quad f_s = \frac{I}{R}\alpha = \frac{I}{R^2}a = \frac{m_2}{2}a$

$\Rightarrow \quad m_2 g\sin\theta - \frac{m_2}{2}a = m_2 a$

$\Rightarrow \quad m_2 g\sin\theta = \frac{3}{2}m_2 a \quad \Rightarrow \quad \boxed{a = \frac{2}{3}g\sin\theta}$

c.



$a_1 = a_2$ if strings taught

x_1: $T + m_1 g\sin\theta - f_k = m_1 a$

y_1: $N_1 - m_1 g\cos\theta = 0$

x_2: $m_2 g\sin\theta - T - f_s = m_2 a$

y_2: $N_2 - m_2 g\cos\theta = 0$

τ: $f_s R = I\alpha$

$a = R\alpha$

↓

so.. combining $\tau \Rightarrow f_s = \frac{m_2}{2} a$

with $x_2 \Rightarrow m_2 g \sin\theta - T = \frac{3}{2} m_2 a$

$\Rightarrow T = m_2 (g \sin\theta - \frac{3}{2} a)$

with $x_1 \Rightarrow m_2 g \sin\theta - \frac{3}{2} m_2 a + m_1 g \sin\theta - \mu_k m_1 g \cos\theta = m_1$

$\Rightarrow (m_1 + m_2) g \sin\theta - \mu_k m_1 g \cos\theta = (m_1 + \frac{3}{2} m_2) a$

$$\Rightarrow a = \frac{(m_1 + m_2) g \sin\theta - \mu_k m_1 g \cos\theta}{m_1 + \frac{3}{2} m_2}$$

Check algebra with limits cases

d. at minimal coefficient μ_s

$f_s = \mu_s N_2 = \mu_s m_2 g \cos\theta$

so $\mu_s m_2 g \cos\theta = \frac{m_2}{2} a$

$$\mu_s = \frac{1}{2 g \cos\theta} \cdot \frac{(m_1 + m_2) g \sin\theta - \mu_k m_1 g \cos\theta}{m_1 + \frac{3}{2} m_2}$$

$\mu_s = .097$

e. well, lets check, from part a $a_{block} = .59 \, m/s^2$

part b $a_{cyl} = 2.2 \, m/s^2$

part c $a_{tot} = 1.8 \, m/s^2$

so it makes sense that taught cylinder pulls down block..

No. 937 811E
Engineer's Computation Pad
STAEDTLER

Last Name: ______________________ First Name: ______________________

Physics 101 Fall 2001: Final—Multiple-Choice Answers
2-5 PM, 12 December, 2001

For each question, mark a single X in the box corresponding to the correct answer.

	A	B	C	D	E
1			X		
2					X
3		X			
4				X	
5				X	
6				X	
7					X
8			X		
9			X		
10		X			
11					X
12			X		
13				X	
14		X			
15	X				