Last Name: First Name:

Physics 101 Fall 2001: Test 1—Free Response and Instructions

- Print your LAST and FIRST name on the front of your blue book, on this question sheet and on the multiple-choice question sheet.
- The test consists of two free-response questions and ten multiple-choice questions.
- The test is graded on a scale of 100 points; the first free-response question accounts for 30 points, the second for 40 points and the multiple-choice questions account for 30 points.
- Answer the two free-response questions in your blue book. Answer the multiple-choice questions by circling directly on the question sheet the single answer that is most nearly correct.
- Consult no books or notes of any kind. You may use a hand-held calculator in non-graphing, non-programmed mode.
- Do NOT take test materials outside of the class at any time. Return this question sheet along with your blue book and multiple-choice question sheet.
- Write and sign the Pledge on the front of your blue book.

Show your work for the free-response problems, including neat and clearly labelled figures, in your blue book. Answers without explanation (even correct answers) will not be given credit.

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1. A 2-kg ball is dropped from a height of $h = 35.0$ m. The wind is blowing and imparts a constant horizontal force of 3 N on the ball.

   (a) How long does it take for the ball to reach the ground? [**6 pts**]
   The general equations of motion are

$$y = y_0 + v_{0y}t + \frac{a_y t^2}{2} \tag{1}$$
$$x = x_0 + v_{0x}t + \frac{a_x t^2}{2}. \tag{2}$$

   For this part, we only need to worry about the vertical component, i.e. eq. (1) because with constant acceleration the x- and y- components are independent. We know that $v_{0y} = 0$, $a_y = -g$, $y_0 = h$ and $y = 0$ (where I've chosen the origin of my y-axis at ground height). Using this information, (1) becomes

$$0 = h - \frac{gt^2}{2}$$
$$\Rightarrow t = \sqrt{\frac{2h}{g}}$$
$$= \underbrace{2.67 \text{ s}} \tag{3}$$

   (b) With what velocity does the ball hit the ground? [**10 pts**]
   There are several ways to do this problem and there are two way to answer it. I'll show both answers, but I'll solve the problem using components.
   First, let me right the equation for the components of the velocity as a function of time:

$$v_x = v_{0x} + a_x t \tag{4}$$
$$v_y = v_{0y} + a_y t. \tag{5}$$

We know all the information to find v_y at the time of impact (we found t in the previous section):

$$\begin{aligned} v_y &= -gt \\ &= -26.2 \text{ m/s}. \end{aligned} \tag{6}$$

To find v_x, first we need to know a_x. Using Newton's second law for the x-components, $a_x = F_{net\ x}/m$. Plugging in the numbers, $a_x = 1.5$ N if we choose the acceleration to be in the positive x-direction. Also, $v_{0x} = 0$ and its still the same time, so

$$\begin{aligned} v_x &= a_x t \\ &= 4.01 \text{ m/s}. \end{aligned} \tag{7}$$

So, then the velocity $\vec{\mathbf{v}} = (4.01 \text{ m/s})\hat{\imath} - (26.2 \text{ m/s})\hat{\jmath}$. Other sign conventions are possible...if you explained them. Another way to express the answer is in polar notation. Then $v = \sqrt{v_x^2 + v_y^2} = 26.5$ m/s and the direction is $\theta = \tan^{-1}(v_y/v_x) = 81.3^o$ from the ground.

(c) Show that the path of a ball (either $y(x)$ or $x(y)$) is a straight line and find the values of R and θ in Fig. 1. [**14 pts**]
Lets find $y(x)$. First I'll write simplified versions of (8) and (9) using what we know:

$$y = h - \frac{1}{2}gt^2 \tag{8}$$

$$x = \frac{1}{2}a_x t^2. \tag{9}$$

If I solve for t^2 in (9), I get

$$t^2 = \frac{2x}{a_x}.$$

Substituting this into (8), I get

$$y = h - \frac{g}{a_x}x. \tag{10}$$

This is the equation of a line with y-intercept h and slope $-g/a_x$. QED
Then there are many ways to find R. We could use (9) and the value of t from part 1.a., but I'll do it differently. When $y = 0$, we can see from (10) that

$$\begin{aligned} x = R &= \frac{a_x}{g}h \\ \Rightarrow R &= \underbrace{5.36 \text{ m}}. \end{aligned} \tag{11}$$

The value of θ can also be found in many ways. Its the same angle I found in part 1.b. and it is also the inverse tangent of the slope of $y(x)$. But perhaps the easiest is since we know R we know

$$\begin{aligned} \tan\theta &= \frac{h}{R} \\ \Rightarrow \theta &= \tan^{-1}(\frac{h}{R}) \\ &= \underbrace{81.3^o} \end{aligned} \tag{12}$$

2. Consider the Phys101 student standing on the wheeled wedge on the ramp of angle θ as depicted in Fig. 2. The wheeled wedge has mass m and the student has mass M. As the wheeled wedge accelerates down the ramp, the student maintains the same position on the wedge. Take the wheels to be very small and frictionless.

    (a) Consider the wedge and student as a single object sliding down a frictionless inclined plane. In terms of some or all of m, M, g and θ, what is the acceleration of the student and wedge down the ramp?[**7 pts**]

Look at figure 2.a for a picture of the free body diagram of the student-wedge combo. The total mass of the the student-wedge combo is $M + m$. There are two forces: the normal force of the ramp on the wedge N and the weight of the wedge-student combo $W_{w+s} = (M + m)g$. Choosing the x-axis to point down the ramp as in figure 2.a, I will apply Newton's second law for both components (getting the factors of sine and cosine from applying geometry to the FBD 2.a.):

$$x : (M + m)g \sin\theta = (M + m)a_x \tag{13}$$

$$y : N - (M + m)\cos\theta = (M + m)a_y. \tag{14}$$

Because the ramp is solid we know $a_y = 0$. So than means that the only acceleration is that in the x-direction, i.e. $a = a_x$. Solving for $a = a_x$ in (13), we get

$$a = \underbrace{g \sin\theta}. \tag{15}$$

(b) Draw a free body diagram showing all the forces acting on the wheeled wedge and give a brief description of each. [**12 pts**]

See figure 2.b. List of forces on wedge:

- N—normal force of ramp on wedge
- n—normal force of student on wedge
- mg—weight of wedge
- f—static frictional force of student on wedge

(c) Draw a free body diagram showing all the forces acting on the student and give a brief description of each. [**9 pts**]

See figure 2.c. List of forces on student:

- n—normal force of wedge on student
- Mg—weight of wedge
- f—static frictional force of wedge on student

(d) In terms of some or all of m, M, g and θ, what is the frictional force of the wedge on the student? [**5 pts**]

The first thing to understand is that if the wedge was frictionless, then the wedge would slip out from under the student and the student would fall on his or her ass. So its the friction that provides the horizontal acceleration of the student. But what is the horizontal acceleration of then student? See figure 2.d for a picture that shows how to break the acceleration into x-component (horizontal) and y-component (vertical).

From the picture we see

$$\begin{aligned} a_x &= a\cos\theta \\ &= g\sin\theta\cos\theta \text{ (using (15))}. \end{aligned} \tag{16}$$

The friction is the only force in the x-direction, so by Newton's second law

$$f = Ma_x = \underbrace{Mg \sin\theta \cos\theta}$$

(e) If, as the wedge accelerates down the ramp, the student is standing on a bathroom spring scale that measures the normal force on the student, what would the scale read in terms of some or all of m, M, g and θ? [**7 pts**]

The question is asking for and expression for n. To get this, we first write Newton's second law for the y-direction:

$$n - Mg = Ma_y.$$

But what is a_y? From figure 2.d, we can see that

$$\begin{aligned} a_y &= -a\sin\theta \\ &= -g\sin^2\theta. \end{aligned} \tag{17}$$

So then

$$\begin{aligned} n &= Mg + Ma_y \\ &= Mg - Mg\sin^2\theta \\ &= Mg(1 - \sin^2\theta) \\ &= \underbrace{Mg\cos^2\theta}. \end{aligned} \tag{18}$$

This answer makes sense because it is less than the weight and the student should feel weightless if $\theta = 90^o$

1. d
2. b
3. a
4. d
5. d
6. b
7. d
8. c
9. d
10. b

Fig. 2.a

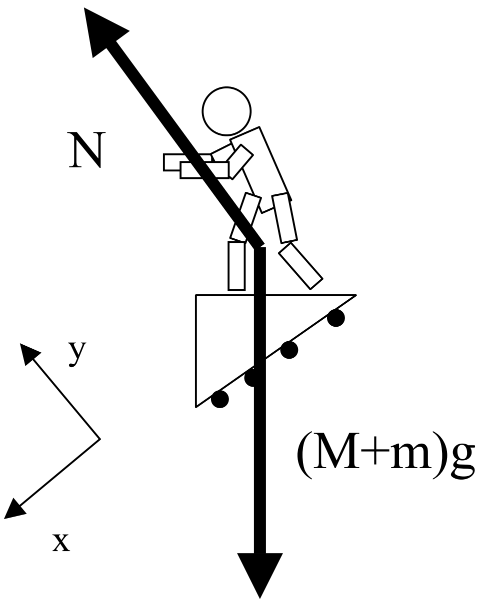


Fig. 2.b

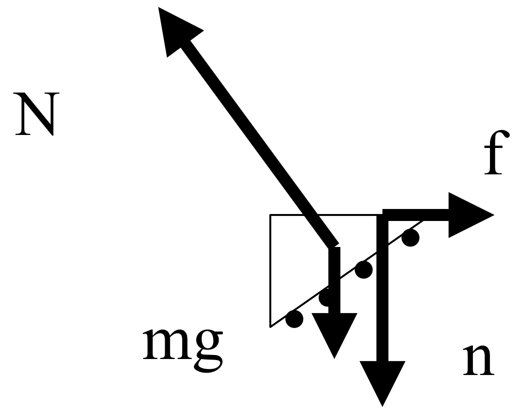


Fig. 2.c

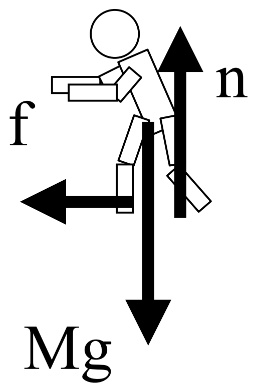
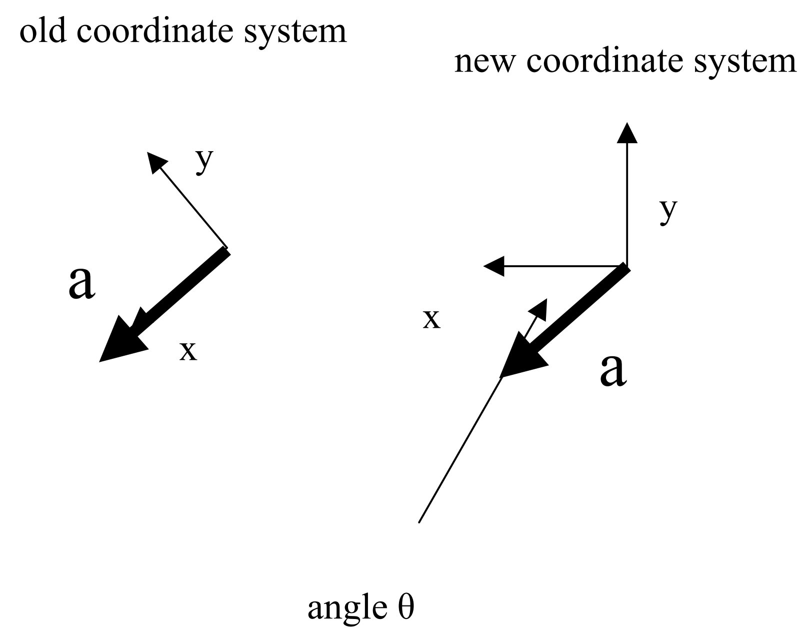


Fig. 2.d



old

$a_x = g \sin\theta$

new

$a_x = -\, g \sin\theta \cos\theta$

$a_y = g \sin^2\theta$