# Solution to Chapter 14 problems

For Phys101

#### December 5, 1998

## **Problem 11: Initial conditions**

This problem is a straight test in using the equations of motion

$$x(t) = A\cos(\omega t + \delta) \tag{1}$$

and its consequences

$$v(t) = -\omega A \sin(\omega t + \delta) \tag{2}$$

and

$$v(t) = -\omega^2 A \cos(\omega t + \delta)$$
(3)

We also have to use the relationships

$$2\pi f = \omega \tag{4}$$

and

$$f = \frac{1}{T}.$$
(5)

We are given the initial conditions x(0), v(0) and T = 1.5s. The first thing is to work out that

$$\omega = \frac{2\pi}{T} = 4.19 \text{ rads/s.} \tag{6}$$

Next, since

$$x(0) = A\cos(\delta) \tag{7}$$

and

$$v(0) = -\omega A \sin(\delta) \tag{8}$$

we have, dividing one equation by the other, that

$$2.0 = -\omega \tan(\delta) = -4.19 \tan(\delta) \tag{9}$$

or that  $\delta = -0.445$  rads. The only thing we do not know is A and we can get that from

$$25 = A\cos(-0.445) \tag{10}$$

or that A = 27.7. Which means that

$$x = 27.7\cos(4.19t - 0.445) \tag{11}$$

and

$$v = -116.06\sin(4.19t - 0.445) \tag{12}$$

and

$$a = -486.30\cos(4.19t - 0.445). \tag{13}$$

## Problem 26: Total energy

The total energy of something oscillating on a spring is

$$E = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2 A^2$$
 (14)

where we have used

$$\omega = \sqrt{k/m}.\tag{15}$$

Of the things we need to compute the energy, in this case we know the amplitude A = 8cm and the mass m and so we need to figure out either the spring constant k or  $\omega$  or get rid of them somehow. We also have another bit of information, the maximum acceleration  $a_m$ .

Well, looking back at the equation for acceleration, we can see that the magnitude of the maximum acceleration must be such that

$$a_m = \omega^2 A \tag{16}$$

so that

$$E = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}ma_m A \tag{17}$$

which we can now compute to get 0.42J.

#### **Problem 42: Gravity and springs**

The first thing do figure out in this problem is to use the initial information that we have been given – that the spring compresses through 3cm when a 2kg mass is put on it.

When the system is at equilibrium, the gravitational and spring forces must equal each other, so this means that

$$kx_0 = mg \tag{18}$$

where  $x_0 = 3$ cm is the compression and so we now know the spring constant

$$k = \frac{mg}{x_0}.$$
(19)

Even though we have not been asked for it, that is clearly going to be useful information!

We also know that the spring in a constant gravitional field behaves exactly like the spring without any gravity (look at the text) provided we remember to measure everything from its new equilibrium position.

We are told that the system is given an initial velocity 0.3m/s when it is at the equilibrium position and are asked about the maximum height to which the mass rises. We could do this by pure energy conservation relations, but let us do it by considering the oscillatory motion.

The mass is going to go downwards and then back up again as it oscillates. The maximum height to which it rises must be when it is at the extreme of its oscillation on the upward part of the cycle. So that means that what we have to do basically is to figure out the amplitude of the oscillation.

Looking back at the equations of motion, we know that in this particular case the angle  $\delta$  must be such that

$$v(t) = -\omega A \cos(\omega t) \tag{20}$$

and

$$x(t) = A\sin(\omega t) \tag{21}$$

(where x is the distance from  $x_0$ ; think of what is zero at t = 0 and what is at its maximum). This means that v(0) = 0.3m/s=  $-\omega A$ . What is  $\omega$ ? We know that it is  $\omega^2 = k/m$  and since we know k from above, we can deduce that

$$\omega = \sqrt{g/x_0} \tag{22}$$

so that

$$A = \frac{-v(0)}{\sqrt{g/x_0}} = 1.66 \text{cm}$$
(23)

so that the maximum height is 5 + 1.66 = 6.66cm.

How long does it take to get there – well, we know that this is at the top of its upwards part of the oscillation. An oscillation consists of equal time spent on the following 1) going down from the equilibrium, 2) coming back up towards the equilibrium, 3) going up away from the equilibrium and 4) coming back down to the equilibrium. This means that the mass took 3/4of the time for an oscillation (the period T) or

$$t = 0.75 \times T = 0.75 \frac{2\pi}{\omega} = 0.261$$
s. (24)

Does the spring become uncompressed ? Well, it needs to get to A = 3 before it gets uncompressed, so no it doesn't.

Finally, what velocity must it have so that it does get uncompressed ? Look at the equation for A above and now solve for v(0) using A = 3.0cm, so that

$$v(0) = 3\sqrt{(g/x_0)} = .542$$
m/s. (25)

### **Problem 65: Physical Pendulum**

The behavior of a physical pendulum is described on pages 419 -421 in the text and this problem is a direct application.

The relationship for the period and the dimensions of the mass is

$$T = 2\pi \sqrt{\frac{I}{Mgd}} \tag{26}$$

where I is the moment of inertia of the body around the point of suspension and d is the distance of suspension. This means that we can use the parallel axis theorem to write  $I = I_{com} + md^2$  and so we get

$$T = 2\pi \sqrt{\frac{0.5Mr^2 + Md^2}{Mgd}}.$$
(27)

If we square both sides we get a quadratic equation in d

$$4\pi^2 d^2 - T^2 g d + 2\pi^2 r^2 \tag{28}$$

which we can solve to get that d = 1.31, 0.244m. Since the disk is only 0.8m in radius, the first answer is unphysical and so we get that d = 0.244m.

To do this part, we have to minimize the equation for T with respect to D. We can just go ahead and rewrite the equation as one for  $T^2$  and minimize that instead – i.e., set

$$T^2 = 2\pi^2 \frac{r^2}{gd} + 4\pi^2 \frac{d}{g}$$
(29)

and then  $d(T^2)/dd = 0$  so that

$$-\frac{r^2}{gd^2} + 2\frac{1}{g} = 0 \tag{30}$$

or that

$$d^2 = \frac{r^2}{2} \tag{31}$$

and so that  $d = \sqrt{r/2}$ .

Substituting this in the equation for T we get that

$$T = 2\pi \sqrt{\frac{r/sqrt2}{g}} = 2.1s.$$
(32)

## Problem 107: Blocks on a spring

The first part is easy: We know how frequencies are related to masses and have been given the total mass and the frequency f so that

$$\omega = 2\pi f = \sqrt{k/(m_1 + m_2)}$$
(33)

but  $m_1 = m_2$  so that this means that

$$m = \frac{k}{8\pi^2 f^2} = 2.35 \text{kg.}$$
(34)

When the blocks slide, this means that the frictional force is unable to provide the acceleration for the upper block at some point (draw a free-body diagram and set the horizontal force equal to the acceleration). This means that when it just slides, the maximum acceleration and the coefficient of static friction must be related by

$$\mu_s N = \mu_s mg = ma_m \tag{35}$$

so that

$$\mu_s = a_m/g. \tag{36}$$

So now we have to figure out  $a_m$ . From earlier problems, we know that  $a_m = A\omega^2 = 4\pi^2 A f^2$  so that we get that

$$\mu_s = \frac{4\pi^2 A f^2}{g} = 0.652. \tag{37}$$

## Problem 119

This one is actually identical to the last part of the previous problem, so that I am not going to solve it. The answers are

$$\mu_s = \frac{4A\pi^2}{T^2g} = 2.52\tag{38}$$

which is an absurd number, and conversely

$$A_m = \frac{\mu_s g}{\omega^2} = 6.36 \text{cm.} \tag{39}$$

### Problem 122: Rock and roll!

This is a hard problem to solve unless we use energy conservation, so we shall do that. What is the total energy of the system ?

$$E = E_{spring} + E_{kinetic}.$$
 (40)

If we think about it for a while, it becomes clear that  $E_{kinetic}$  is itself  $E_{linear} + E_{rolling}$  so that we get that

$$E = \frac{1}{2}kx^{2} + \frac{1}{2}mv^{2} + \frac{1}{2}I\omega_{r}^{2}$$
(41)

where we have used  $\omega_r$  for the rolling motion (this is not the  $\omega$  of the spring moving back and forth that we are looking for, though they must be related!). For this problem, since the cylinder is rolling without slipping,  $\omega_r = v/R$  so that we can get rid of it; also, the moment of inertia for the cylinder is  $I = 1/2mR^2$  so that we get that

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 + \frac{1}{4}mv^2.$$
(42)

Okay, where can we evaluate this quantity? Well, at the equilibrium position of the oscillation, when x = 0, the velocity is at a maximum, so we have

$$E_1 = \frac{3}{4}mv_{max}^2.$$
 (43)

At the other end, when the velocity is zero, the spring is stretched to its maximum amplitude so that we have

$$E_2 = \frac{1}{2}kA^2.$$
 (44)

Since this is a conservative system, we can set the two energies equal to each other to get that

$$kA^2 = \frac{3}{2}mv_{max}^2 \tag{45}$$

and now we use that the maximum velocity is  $\omega A$  to get rid of it so that we get finally that

$$kA^2 = \frac{3}{2}m\omega^2 A^2 \tag{46}$$

and therefore that

$$\omega = \sqrt{\frac{2k}{3M}} = 21.08 \text{rads/s.} \tag{47}$$

What about the slipping part ? Well, we want that the friction just provides the maximum acceleration  $a_m$  so that

$$\mu_s mg = ma_m. \tag{48}$$

The maximum acceleration is such that  $ma_m = kA$  and we know that  $E = \frac{1}{2}kA^2$  so that

$$\sqrt{2Ek} = kA = ma_m \tag{49}$$

so that

$$\mu_s = \frac{\sqrt{2Ek}}{mg} = 3.4. \tag{50}$$