Time allowed: 2 hours in one sitting
Due: Monday, November 5 at 5PM in the box marked Phys101/102 in the Physics Lounge
You may use your own textbook, your notes and a non-programmed calculator. For the purposes of this problem set, you may also use the online solutions to the corresponding suggested problems. You should consult no other help. Please follow the standard format:

- Write legibly on one side of 8-1/2" white or lightly tinted paper.
- Staple all sheets (including this one) together in the upper left corner
- **Make one vertical fold.**
- On the outside (staple side up) on successive lines
    - PRINT your last name in CAPITAL letters.
    - PRINT your first name.
    - Print the phrase 'Pledged Problems 8' and the due date.
    - Print the times at which you started and finished the problems.
    - Write and sign the Pledge, with the understanding you may consult the resources described above.

1. An object at rest on a flat, horizontal surface explodes into two fragments, one seven times as massive as the other. The heavier fragment slides 8.2 m before stopping. How far does the the lighter fragment slide? Assume that both fragments have the same coefficient of kinetic friction.

   SOLUTIONS

   Since there is an external force (friction) acting along the x-direction (horizontal direction), the x-component of momentum is NOT conserved for the system of two fragments, but at the instant the two pieces begin to fly apart, the momentum (x-component) is conserved.

   $$p_{x0} = p_{xf} \tag{1}$$

   $$0 = 7\,m\,v_M - m\,v_m \tag{2}$$

   $$v_M = \frac{v_m}{7} \tag{3}$$

   Since friction decelerates the masses to zero speed, we can calculate the *NET* work on the individual blocks and relate this quantity to the change in kinetic energy of each block.

   $$W_{NET,7m} = -\mu_k\,M\,g\,(d_1) = -\frac{1}{2}\,(7\,m)\,v_M^2 \tag{4}$$

   $$v_M^2 = 2\,\mu_k\,g\,d_1 \tag{5}$$

   $$W_{NET,m} = -\mu_k\,m\,g\,(d_2) = -\frac{1}{2}\,m\,v_m^2 \tag{6}$$

   $$v_m^2 = 2\,\mu_k\,g\,d_2 \tag{7}$$

   $$\Rightarrow d_1 = \frac{1}{49}\,d_2 \tag{8}$$

   $$\boxed{d_2 = 49\,d_1 = 401.8 \text{ m}}\ \checkmark \tag{9}$$

2. A fireworks rocket is launched vertically upward with an initial speed of 40 m/s. At the peak of its trajectory, it explodes into two equal-mass fragments. One fragment reaches the ground 2.87 s after the explosion. When does the second fragment reach the ground?

SOLUTION

Since the firework rocket is launched vertically upward, we need to calculate the maximum height (relative to the ground) the rocket achieves.

$$y_f = y_0 + v_{0y}\,\Delta t - \frac{1}{2}\,g\,\Delta t^2 \tag{10}$$

$$(\text{or Work-Energy}) \; -m\,g\,y_f = -\frac{1}{2}\,m\,v_0^2 \tag{11}$$

$$y_f = \frac{v_0^2}{2\,g} = 81.55m \quad (\text{Max. height before explosion.}) \tag{12}$$

The momentum of the system is NOT conserved since gravity is acting on the system. Recalling the behavior of a projectile, the y-component of velocity (and hence momentum) will determine the flight times. (Assume that $v_{1,y}$ points upward and that $v_{2,y}$ is directed toward the earth.)

The time it takes a massive body to fall to the earth from an initial height of 81.55 m is 4.1 s (verify this number); therefore, the 2.87s corresponds to the time it takes a projectile thrown down toward the earth to reach the earth.

$$p_{f,y} = p_{0,y} \quad (\textbf{\textit{Immediately}} \text{ before and after the explosion.}) \tag{13}$$

$$m\,v_{1,y} - m\,v_{2,y} = 0. \tag{14}$$

$$v_{1,y} = v_{2,y} \tag{15}$$

$$y_f = y_0 + v_{2,y}\,\Delta t - \frac{1}{2}\,g\,\Delta t^2 \tag{16}$$

$$v_{2,y} = 14.33m/s \tag{17}$$

$$\Rightarrow v_{1,y} = 14.33m/s \quad (\text{away from the earth}) \tag{18}$$

The time it takes a projectile to return back to its original launch point assuming the projectile was launched vertically with speed v is $t = \frac{2\,v}{g} = 2.92$ s. Once the projectile reaches its original launch point, it now has a vertical speed equal to v pointing down (toward the earth). We know from the statement of the problem that it takes 2.87s for this projectile to hit the earth. The total time it takes the second mass to reach the ground is $\boxed{T = 2.92s + 2.87s = 5.79s}$.

3. A cookie sheet moves along a conveyor belt towards an oven, with mounds of unbaked cookie dough dropping vertically onto it at the rate of one 12-gram mound every two seconds. What average force must the conveyor belt exert on the cookie sheet to keep it moving at a constant speed of 50 cm/s?

SOLUTION

To find the average force the conveyor belt exerts on the cookie sheet to keep it moving at a constant speed, first consider the average force exerted on the cookie dough. Newton's third law ensures that the magnitude of the force exerted on the cookie dough by the cookie sheet is equal to the magnitude of the force exerted on the cookie sheet by the cookie dough. The conveyor belt will have to balance this force in order to keep the sheet moving a constant speed.

$$F_{avg,x} = \frac{\Delta p_x}{\Delta t} \tag{19}$$

$$\Delta p_{x,dough} = \Delta p_{x,sheet} \tag{20}$$

$$\Delta p_{x,dough} = m_{dough} \left(0.5m/s - 0\right) \tag{21}$$

$$m_{dough} = 0.006kg/s \times \Delta t. \tag{22}$$

$$F_{avg,x} = 0.006 \left(0.5\right) N = 3.0 \times 10^{-3} \, N \tag{23}$$