Physics 101 Fall 2007: Pledged Problems 7 – Energy Principles II

Time allowed: `2 hours in one sitting`
Due: `Monday, October 29 at 5PM in the box marked Phys101/102 in the Physics Lounge`
You may use your own textbook, your notes and a non-programmed calculator. For the purposes of this problem set, you may also use the online solutions to the corresponding suggested problems. You should consult no other help. Please follow the standard format:

- Write legibly on one side of 8-1/2" white or lightly tinted paper.
- Staple all sheets (including this one) together in the upper left corner
- **Make one vertical fold.**
- On the outside (staple side up) on successive lines
  - PRINT your last name in CAPITAL letters.
  - PRINT your first name.
  - Print the phrase 'Pledged Problems 7' and the due date.
  - Print the times at which you started and finished the problems.
  - Write and sign the Pledge, with the understanding you may consult the resources described above.

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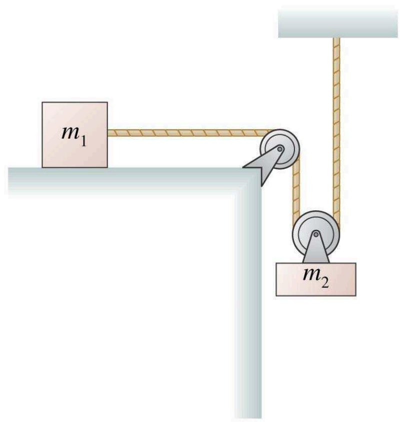
1. Two blocks labeled m_1 and m_2 are connected together by a light rope and pulley assembly as shown in the figure below.



(a) Assume there is NO friction between block m_1 and the floor. If the system is released from rest, what is the speed of block m_1 at the moment block m_2 has descended a vertical distance y from its initial position?

Solution

Since there is no friction, the *net* work on the system of the two blocks is the work due to gravity.

$$W_{net} = m_2\, g\, y \tag{1}$$

$$W_{net} = \Delta K = \frac{1}{2}\, m_1\, v_1^2 + \frac{1}{2}\, m_2\, v_2^2 \tag{2}$$

$$m_2\, g\, y = \frac{1}{2}\left(m_1\, v_1^2 + m_2\, v_2^2\right) \tag{3}$$

Since the two masses are connected a compound pulley system, we need to find the relationship between v_1 and v_2. Use *conservation of string* to solve for the relationship.

$$v_1 + 2\,v_2 = 0 \tag{4}$$

$$\Rightarrow m_2\,g\,y = \frac{1}{2}\left(4\,m_1\,v_2^2 + m_2\,v_2^2\right) \tag{5}$$

$$\Rightarrow v_2 = \sqrt{\frac{2\,m_2\,g\,y}{(4\,m_1 + m_2)}} \tag{6}$$

$$\Rightarrow v_1 = \sqrt{\frac{8\,m_2\,g\,y}{(4\,m_1 + m_2)}} \tag{7}$$

(b) Assume there is friction between m_1 and the floor with a coefficient of kinetic friction μ_k. Assume also that the system is released from rest and that the weight of block m_2 is sufficient to cause block m_1 to begin sliding across the floor. What is now the speed of block m_2 at the moment block m_2 has descended a vertical distance y from its original position?

Solution

Now there is friction; therefore, the *net* work on the system of the two blocks is the work due to gravity plus the work due to friction.

$$W_{net} = m_2\,g\,y - \mu_k\,m_1\,g\,x_1 \tag{8}$$

$$W_{net} = \Delta\,K = \frac{1}{2}\,m_1\,v_1^2 + \frac{1}{2}\,m_2\,v_2^2 \tag{9}$$

$$m_2\,g\,y - \mu_k\,m_1\,g\,x_1 = \frac{1}{2}\left(m_1\,v_1^2 + m_2\,v_2^2\right) \tag{10}$$

Using conservation of string:

$$x_1 = 2\,y \tag{11}$$

$$v_1 + 2\,v_2 = 0 \tag{12}$$

$$\Rightarrow m_2\,g\,y - 2\,\mu_k\,m_1\,g\,y = \frac{1}{2}\left(4\,m_1\,v_2^2 + m_2\,v_2^2\right) \tag{13}$$

$$\Rightarrow v_2 = \sqrt{\frac{2(\,m_2\,g\; - 2\,\mu_k\,m_1\,g)\,y}{(4\,m_1 + m_2)}} \tag{14}$$

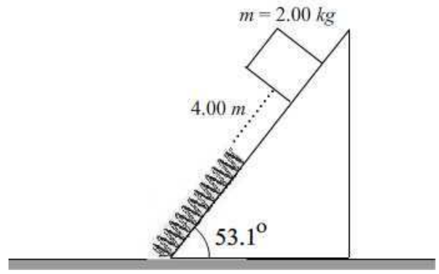
$$\Rightarrow v_1 = \sqrt{\frac{8\,(\,m_2\,g\; - 2\,\mu_k\,m_1\,g)\,y}{(4\,m_1 + m_2)}} \tag{15}$$

2. A 2.00 kg package is released on a 53.1° incline , 4.00 m from a long spring with force constant 120 N/m that is attached at the bottom of the incline as shown in the figure below. The coefficients of friction between the package and the incline are $\mu_s = 0.40$ and $\mu_k = 0.20$. The mass of the spring is negligible.

   (a) What is the speed of the package just before it reaches the spring?

   (b) What is the maximum compression of the spring?

   (c) The package rebounds back up the incline. How close does it get to its initial position?



Solution

a. As the block moves down the incline, friction is doing (negative) work on the block and gravity is doing (positive) work on the block. Will the block even slide down the incline? The *maximum* force due to static friction is less than the force of gravity (down the incline) so the block will slide down the incline. This would have been an awesome problem if the block would not have slid down the incline. Since the block is *sliding* down the incline, we need to use kinetic friction when calculating the amount of work (net) on the block.

$$\Delta K_T + \Delta U_T = \Delta E_T \tag{16}$$

$$\Delta E_T = \int \vec{\mathbf{F_f}} \cdot \vec{\mathbf{ds}} = -F_f\, l \tag{17}$$

$$\frac{1}{2} m \left(v_f^2 - v_0^2 + m\, g\, (y_f - y_0)\right) = -F_f\, l \tag{18}$$

$$v = \sqrt{2\, g\, l\, (\sin\theta - \mu_k \cos\theta)} \tag{19}$$

$$\boxed{v = 7.3 \text{ m/s}}. \tag{20}$$

b. To find the maximum compression of the spring, I will use the change in kinetic energy, change in potential energy and the work done by friction. Starting from the top of the incline (the initial and final kinetic energy of the block is zero):

$$\Delta K_T + \Delta U_T = \Delta E_T \tag{21}$$

$$\Delta E = -F_f\, \Delta s \tag{22}$$

$$m\, g\, (y_f - y_0) + \frac{1}{2} k\, (\Delta s_f^2 - \Delta s_0^2) = -F_f\, \Delta s \tag{23}$$

With $y_f - y_0 = -(l + \Delta d)\sin\theta$; $\Delta s = l + \Delta d$; $\Delta s_f = \Delta d$; and $\Delta s_0 = 0$.

$$\frac{1}{2} k\, \Delta d^2 - \Delta d\, (m\, g \sin\theta - \mu_k\, m\, g \cos\theta) + (\mu_k\, m\, g\, l \cos\theta - m\, g\, l \sin\theta) = 0. \tag{24}$$

$$\boxed{\Delta d = 1.06 \text{ m}} \tag{25}$$

c. Starting with the spring at its maximum compression, let's find out how high the block rebounds up the incline (final and initial kinetic energy is zero).

$$m\,g\,(y_f - y_0) + \frac{1}{2}\,k\,(\Delta s_f^2 - \Delta s_0^2) = -F_f\,\Delta s \tag{26}$$

With $y_f - y_0 = (l' + \Delta d)\,\sin\theta$; $\Delta s = l' + \Delta d$; $\Delta s_f = 0$; and $\Delta s_0 = \Delta d$.

$$\frac{1}{2}\,k\,\Delta d^2 + m\,g\,(\Delta d + l')\,\sin\theta = -\mu_k\,m\,g\,\cos\theta\,(\Delta d + l') \tag{27}$$

$$l' = \frac{k\,\Delta d^2}{2\,(m\,g\,\sin\theta\,+\,\mu_k\,m\,g\,\cos\theta)} - \Delta d \tag{28}$$

$$l' = 2.68m \quad \text{where } l' \text{ is the distance up the incline} \tag{29}$$

$$4m - 2.68m = 1.32m. \tag{30}$$

So the block ends up 1.32 m lower (along the incline) than its original position.