Physics 101 Fall 2007: Pledged Problems 6 – Energy

Time allowed: `2 hours in one sitting`
Due: `Monday, October 22 at 5PM in the box marked Phys101/102 in the Physics Lounge`
You may use your own textbook, your notes and a non-programmed calculator. For the purposes of this problem set, you may also use the online solutions to the corresponding suggested problems. You should consult no other help. Please follow the standard format:

- Write legibly on one side of 8-1/2" white or lightly tinted paper.
- Staple all sheets (including this one) together in the upper left corner
- **Make one vertical fold.**
- On the outside (staple side up) on successive lines
  - PRINT your last name in CAPITAL letters.
  - PRINT your first name.
  - Print the phrase 'Pledged Problems 6' and the due date.
  - Print the times at which you started and finished the problems.
  - Write and sign the Pledge, with the understanding you may consult the resources described above.

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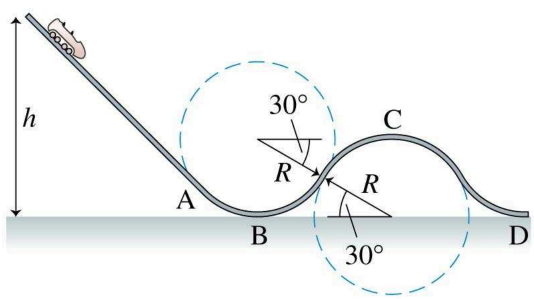
1. A roller coaster car on a frictionless track shown in the figure below starts at rest at height h. The track is straight until point A. Between points A and D, the track consists of circle-shaped segments of radius R. What is the *maximum* height h_{max} from which the car can start so as not to fly off the track when going over the hill at point C? Give your answer in terms of the radius R.



Solution.

If the roller coaster car is moving too fast at point C, the car will fly off the track ($N \to 0$). Applying Newton's 2^{nd} law to the car at point C will give information pertaining to the normal force N.

$$N - m\,g = -\frac{m\,v^2}{R} \tag{1}$$

$$N \to 0 \quad \text{(The car just leaves the track.)} \tag{2}$$

$$m\,g = \frac{m\,v_{max}^2}{R} \tag{3}$$

$$\Rightarrow v_{max} = \sqrt{g\,R} \tag{4}$$

Since we were asked to find the maximum height from which the car should be released from ($v = 0$), we need to consider the work done by gravity on the car. We neglect the normal force's contribution because the normal force is always perpendicular to the car's displacement; hence, the normal force does no work on the car.

$$W_{NET} = \Delta KE \tag{5}$$

$$W_{NET} = W_{gravity} = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} \tag{6}$$

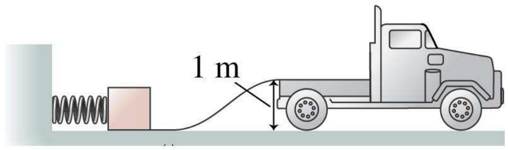
$$W_{gravity} = m\,g\,(h_{max} - R) \quad \text{(NOTE: The work done by gravity is positive.)} \tag{7}$$

$$\Delta KE = m\,g\,(h_{max} - R) \tag{8}$$

$$\frac{1}{2}\,m\,v_{max}^2 = m\,g\,(h_{max} - R) \tag{9}$$

$$\Rightarrow h_{max} = \frac{3}{2}\,R \quad \checkmark \tag{10}$$

2. A freight company uses a compressed spring to launch a 2-kg package up a 1-m high frictionless ramp into a truck, as illustrated in the figure below. The spring constant k is 500 N/m and the spring is initially compressed 30 cm. If the package is released from rest, what is its speed immediately upon entering the truck, i.e., at the top of the ramp?



Solution.

First, find how many forces are doing work on the box. There are two forces doing work on the box: spring forces (vary with position), and gravitational forces.

$$W_{NET} = \Delta\, KE \tag{11}$$

$$W_{NET} = W_{spring} + W_{gravity} = \int \vec{\mathbf{F_s}} \cdot d\vec{\mathbf{x}} + \int \vec{\mathbf{F_g}} \cdot d\vec{\mathbf{y}} \tag{12}$$

$$W_{spring} = \int_{\Delta x}^{0} -k\, x\, dx = \frac{1}{2}\, k\, \Delta x^2 (\text{ The work by the spring is positive.}) \tag{13}$$

$$W_{gravity} = -\, m\, g\, (1m) \text{ (NOTE: The work done by gravity is negative.)} \tag{14}$$

$$\Delta\, KE = -\, m\, g\, (1m) + \frac{1}{2}\, k\, \Delta x^2 \tag{15}$$

$$\frac{1}{2}\, m\, v^2 = -\, m\, g\, (1m) + \frac{1}{2}\, k\, \Delta x^2 \tag{16}$$

With $m = 2$ kg, $\Delta x = 0.3$ m, and $k = 500$ N/m the speed of the of the box at the top of the ramp is

$$\Rightarrow v = \sqrt{\frac{45}{2} - 2\, g} \tag{17}$$

$$\Rightarrow v = 1.7 m/s \quad \checkmark \tag{18}$$