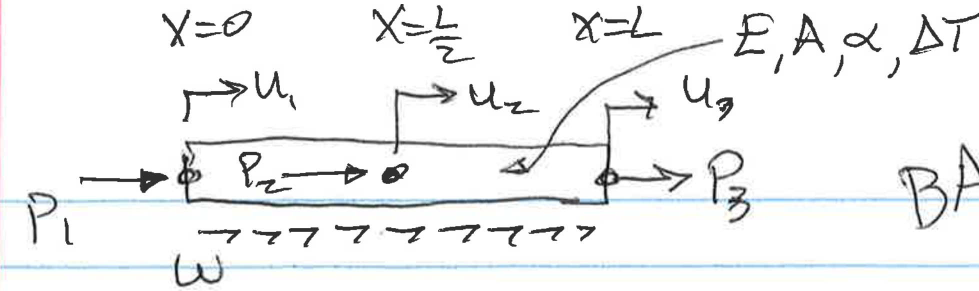
BARS



If $P_2 \neq 0$ then an element interface is required there. Thus, the element matrices are

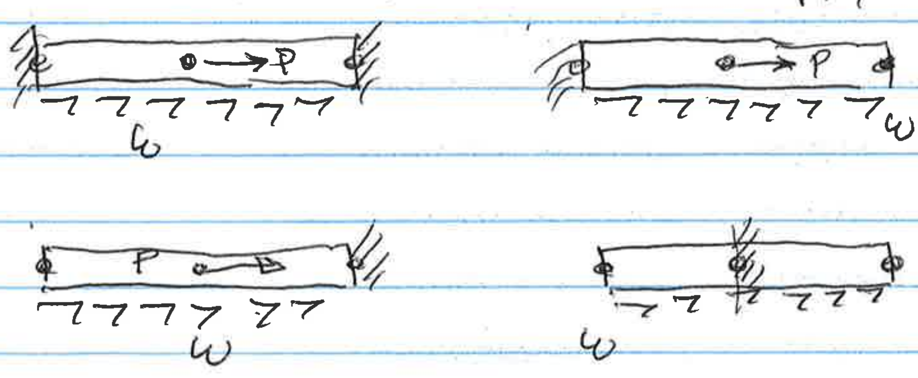
$$\underset{2\times2}{\left[S^e\right]} = \frac{EA}{L^e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad \underset{1\times2}{\left\{C^e_\omega\right\}} = \frac{\omega L}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$\left\{C^e_{\Delta T}\right\} = EA\alpha\Delta T\begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

the two assembled elements, for $L^e = L/2$, are (before BC)

$$\frac{EA}{L^e}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} + \frac{\omega L^e}{2}\begin{Bmatrix} 1 \\ 2 \\ 1 \end{Bmatrix} + EA\alpha\Delta T\begin{Bmatrix} -1 \\ 0 \\ 1 \end{Bmatrix}$$

How many ways can we apply BCs?



? Is the stress tension (+) or compression?

The axial strain in an element is

$$\epsilon = \frac{du}{dx} = \left[\frac{dH(r)}{dx}\right]\{u^e\} \text{. For L2 \& L3}$$

$$\epsilon(r) = \frac{1}{L^e}\begin{bmatrix}-1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\u_2\end{Bmatrix}^e$$

$$\epsilon(r) = \frac{1}{L^e}\begin{bmatrix}(4r-3) & (4-8r) & (4r-1)\end{bmatrix}\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}$$

The axial stress (force per unit area) is

$\sigma(r) = E\,\epsilon(r)$, E = modulus of elasticity

The axial force at a section is

$F(r) = \sigma(r)\,A(r)$, A = cross-sectional area

Example 1 Both ends fixed, P_2 in the middle, $w=0$, $\Delta T=0$. The P_1 and P_3 become reactions. Apply the BCs: Only u_2 is free. $L^e = L/2$

$$\frac{EA}{L/2}[2]\{u_2\} = \{P_2\} + \{0\} + \{0\} - \cancelto{0}{u_1}\{-1\}\frac{EA}{L/2} - \cancelto{0}{u_3}\{-1\}\frac{EA}{L/2}$$

$$\{u_2\} = \left\{\frac{PL/2}{2EA}\right\} = \left\{\frac{PL}{4EA}\right\}$$

Recover reaction P_1 (top row)

$$\frac{EA}{L/2}\begin{bmatrix}1 & -1 & 0\end{bmatrix}\begin{Bmatrix}0\\ \frac{PL}{4EA}\\ 0\end{Bmatrix} = \{P_1\} + \{0\} + \{0\}$$

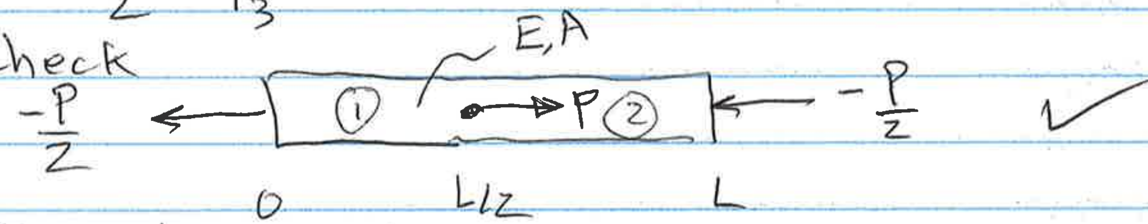
$$-\frac{2EA}{L}\left(\frac{PL}{4EA}\right) = \boxed{\frac{-P}{2} = P_1}$$ ie in neg. x-dir

Reaction P_3 (bottom row)

$$\frac{2EA}{L}\begin{bmatrix}0 & -1 & 1\end{bmatrix}\begin{Bmatrix}0\\ \frac{PL}{4EA}\\ 0\end{Bmatrix} = \{P_3\} + \{0\} + \{0\}$$

$$-\frac{P}{2} = P_3$$

Check



Element strains ; $L^e = L/2$

$$e=1 \quad \epsilon^{(1)} = \frac{1}{L^e}\begin{bmatrix}-1 & 1\end{bmatrix}\begin{Bmatrix}0\\ \frac{PL}{4EA}\end{Bmatrix} = \frac{P}{2EA} \quad \text{(tension)}$$

$$e=2 \quad \epsilon^{(2)} = \frac{2}{L}\begin{bmatrix}-1 & 1\end{bmatrix}\begin{Bmatrix}\frac{PL}{4EA}\\ 0\end{Bmatrix} = \frac{-P}{2EA} \quad \text{(compress)}$$

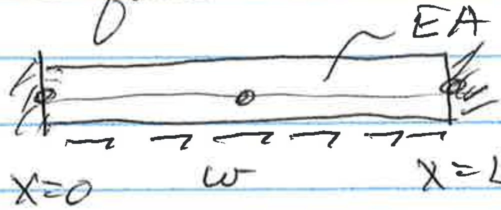
Element stress, $\sigma = E\epsilon$

$$\sigma^{(1)} = \frac{+P}{2A}, \quad \sigma^{(2)} = \frac{-P}{2A}$$

Example 2 Both ends fixed, $\Delta T = 0$, $P_2 = 0$

w = line load per unit length

Use one quadratic element, $L^e = L$



Before BC

$$\frac{EA}{3L^e}\begin{bmatrix} 7 & -8 & 1 \\ -8 & 16 & -8 \\ 1 & -8 & 7 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ \cancel{P_2}^{\,0} \\ P_3 \end{Bmatrix} + \frac{wL^e}{6}\begin{Bmatrix} 1 \\ 4 \\ 1 \end{Bmatrix} + \cancel{EA\alpha\Delta T}^{\,0}\begin{Bmatrix} -1 \\ 0 \\ 1 \end{Bmatrix}$$

Set $u_1 = u_3 = 0$

$$\frac{EA}{3L}[16]\{u_2\} = \{0\} + \frac{wL}{6}\{4\} + \{0\} - \cancel{\frac{u_1}{3L}}^{\,0}\{-8\} - \cancel{\frac{u_3}{3L}}^{\,0}\{-8\}$$

$$\{u_2\} = \left\{\frac{4wL}{6EA}\right\}\frac{3L}{16} = \left\{\frac{wL^2}{8EA}\right\}$$

Interpolate displacement

$$u(r) = [H(r)]\{u^e\}$$

$$= [H_1 \quad H_2 \quad H_3]\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = 0 + H_2(r)u_2 + 0$$

$$u(r) = (4r - 4r^2)\left(\frac{wL^2}{8EA}\right)$$

Check

$r = 0$, $u(0) = u_1 = 0$, $r = 1$, $u(L) = u_3 = 0$ ✓

Element strain

$$\epsilon = \frac{du}{dx} = \frac{1}{L^e}\frac{du}{dr} = \frac{1}{L^e}\left[\frac{dH(r)}{dr}\right]\{u^e\}$$

$$e=1 \quad \epsilon(r) = \frac{1}{L^e}\left[\frac{dH_1}{dr} \quad \frac{dH_2}{dr} \quad \frac{dH_3}{dr}\right]\{u^e\}$$

$$\epsilon(r) = \frac{1}{L^e}\left(0 + \frac{dH_2}{dr}u_2^e + 0\right)$$

$$\epsilon(r) = \frac{1}{L}(4-8r)\frac{\omega L^2}{8EA}$$