Significant Figures in Measurements and Calculations

Understanding Uncertainty (taken from http://chemistry.about.com Nov. 1, 2007)

Every measurement has a degree of uncertainty associated with it. The uncertainty derives from the measuring device and from the skill of the person doing the measuring.

Let's use volume measurement as an example. Say you are in a chemistry lab and need 7 mL of water. You could take an unmarked coffee cup and add water until you think you have about 7 milliliters. In this case, the majority of the measurement error is associated with the skill of the person doing the measuring. You could use a beaker, marked in 5 mL increments. With the beaker, you could easily obtain a volume between 5 and 10 mL, probably close to 7 mL, give or take 1 mL. If you used a pipette marked to with 0.1 mL, you could get a volume between 6.99 and 7.01 mL pretty reliably. It would be untrue to report that you measured 7.000 mL using any of these devices, because you didn't measure the volume to the nearest microliter. You would report your measurement using significant figures. These include all of the digits you know for certain plus the last digit, which contains some uncertainty.

Significant Figure Rules

- Non-zero digits are always significant.
- All zeros between other significant digits are significant.
- The number of significant figures is determined starting with the leftmost non-zero digit. The leftmost non-zero digit is sometimes called the *most significant digit* or the *most significant figure*. For example, in the number 0.004205 the '4' is the most significant figure. The lefthand '0's are not significant. The zero between the '2' and the '5' is significant.
- The rightmost digit of a decimal number is the *least significant digit* or *least significant figure*. Another way to look at the least significant figure is to consider it to be the rightmost digit when the number is written in scientific notation. Least significant figures are still significant! In the number 0.004205 (which may be written as 4.205×10^{-3}), the '5' is the least significant figure. In the number 43.120 (which may be written as 4.3210×10^{1}), the '0' is the least significant figure.
- If no decimal point is present, the rightmost non-zero digit is the least significant figure. In the number 5800, the least significant figure is '8'.

Uncertainty in Calculations

Measured quantities are often used in calculations. The precision of the calculation is limited by the precision of the measurements on which it is based.

- **Addition and Subtraction**
  When measured quantities are used in addition or subtraction, the uncertainty is determined by the absolute uncertainty in the least precise measurement (not by the number of significant figures). Sometimes this is considered to be the number of digits after the decimal point. Example
  32.01 m
  5.325 m
  12 m

  Added together, you will get 49.335 m, but the sum should be reported as '49' meters.
- **Multiplication and Division**
  When experimental quantities are mutiplied or divided, the number of significant figures in the result is the same as that in the quantity with the smallest number of significant figures. If, for example, a density calculation is made in which 25.624 grams is divided by 25 mL, the density should be reported as 1.0 g/mL, not as 1.0000 g/mL or 1.000 g/mL.

Losing Significant Figures

Sometimes significant figures are 'lost' while performing calculations. For example, if you find the mass of a beaker to be 53.110 g, add water to the beaker and find the mass of the beaker plus water to be 53.987

g, the mass of the water is 53.987-53.110 g = 0.877 g
The final value only has three significant figures, even though each mass measurement contained 5 significant figures.

Rounding and Truncating Numbers

There are different methods which may be used to round numbers. The usual method is to round numbers with digits less than '5' down and numbers with digits greater than '5' up (some people round exactly '5' up and some round it down).

Example:
If you are subtracting 7.799 g - 6.25 g your calculation would yield 1.549 g. This number would be rounded to 1.55 g, because the digit '9' is greater than '5'.

In some instances numbers are truncated, or cut short, rather than rounded to obtain appropriate significant figures. In the example above, 1.549 g could have been truncated to 1.54 g.

Exact Numbers

Sometimes numbers used in a calculation are exact rather than approximate. This is true when using defined quantities, including many conversion factors, and when using pure numbers. Pure or defined numbers do not affect the accuracy of a calculation. You may think of them as having an infinite number of significant figures. Pure numbers are easy to spot, because they have no units. Defined values or conversion factors, like measured values, may have units. Practice identifying them!

Example:
You want to calculate the average height of three plants and measure the following heights: 30.1 cm, 25.2 cm, 31.3 cm; with an average height of (30.1 + 25.2 + 31.3)/3 = 86.6/3 = 28.87 = 28.9 cm. There are three significant figures in the heights; even though you are dividing the sum by a single digit, the three significant figures should be retained in the calculation.

Accuracy and Precision

Accuracy and precision are two separate concepts. The classic illustration distinguishing the two is to consider a target or bullseye (we'll use arrows in this example). Arrows surrounding the bullseye indicate a high degree of accuracy; arrows very near to each other (possibly nowhere near the bullseye) indicate a high degree of precision. To be accurate an arrow must be near the target; to be precise successive arrows must be near each other. Consistently hitting the very center of the bullseye indicates both accuracy and precision.

Consider a digital scale. If you weigh the same empty beaker over and over and over again the scale will yield values with a high degree of precision (say 135.776 g, 135.775 g, 135.776 g). The actual mass of the beaker may be very different. Scales (and other instruments) need to be calibrated! Instruments typically provide very precise readings, but accuracy requires calibration. Thermometers are notoriously inaccurate, often requiring re-calibration several times over the lifetime of the instrument. Scales also require recalibration, especially if they are moved or mistreated.

Do you need more information and examples about significant figures? There are several resources available on the internet.

Additional Reading

- Significant Figure Tutorial - Do you think you have a handle on significant figures? This site offers a quick self-test.
- Significant Figures - This site offers a comprehensive discussion of significant figures. There are separate sections for defining significant figures, telling which numbers are significant, dealing with

zeros, understanding scientific notation, making calculations, reviewing concepts, making measurements, introducing additional principles, and working problems.

- Introduction to Measurements and Uncertainty - This interactive online tutorial covers measurement and uncertainty as it relates to length, temperature, volume, and exact numbers and also discusses the counting of significant figures. There are sections on moving decimal points, counting zeros, and rounding. There is an opportunity to test your understanding of each concept.
- Significant Figure Rules - This page provides a quick reference for working with significant figures, including a summary of rules used during addition/subtraction and multiplication/division.