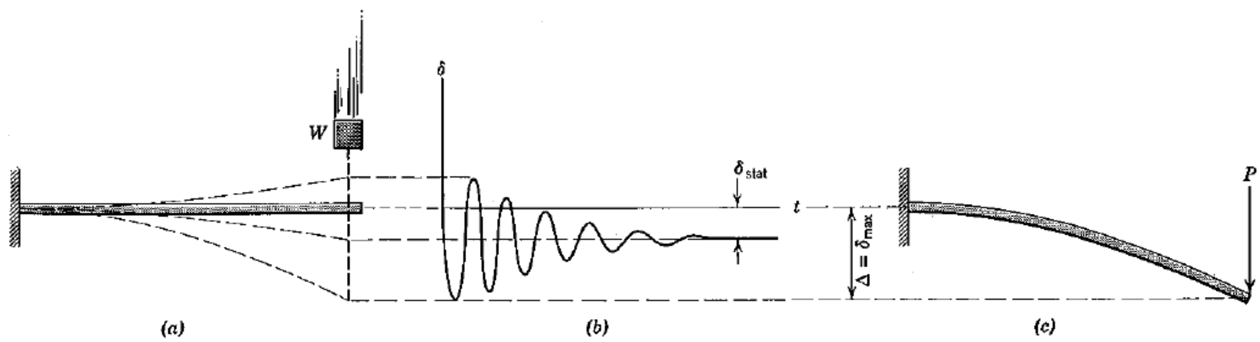
Impact Load Factors for Static Analysis

Often a designer has a mass, with a known velocity, hitting an object and thereby causing a suddenly applied impact load. Rather than conduct a dynamic analysis an amplified static analysis is used, at least for the preliminary design. In a static stress analysis the static force (or weight of the mass) must be increased by an “Impact Factor” so as to obtain a good approximation of the maximum dynamic deflection and stress. For hard elastic bodies the Impact Factor, for vertical impacts, is always greater than or equal to two. It is not unusual for the vertical Impact Factor to be more than 10, 100, or more than 1,000. The following figure shows an amplified static load, P, used to estimate the maximum deflection and stress in a beam due to the dynamic response of a cantilever beam having a weight, W, dropped vertically onto it.



A static force P approximates the dynamic effects of a falling weight W

In the study of the mechanics of solids, an energy balance approximation is used to estimate the required static load. That approximation assumes that all of the kinetic energy of the moving mass is converted, with an efficiency of η, to strain energy in the body. If you assume that no noise, or heat, or inelastic response, and neglect the mass of the struck member then $\eta = 1$ and the collision is 100% efficient. There are several handbook equations that provide corrections to the elementary theory based on the ratio of the striking mass to the member mass. Those corrections seldom give an efficiency of less than $\eta = 0.95$.

In the case of a weight dropped vertically from a height, *h*, the vertical Impact Factor is

$$n = 1 + \sqrt{1 + \frac{2h\eta}{\delta_{static}}}$$

where δ_{static} is the deflection of the member due to a static force (W) applied at the impact point in the impact direction. So, even if *h = 0* this factor is equal to two. Similarly, a mass moving horizontally with a velocity of *v* has an horizontal Impact Factor of

$$n = \sqrt{\frac{\eta v^2}{g\,\delta_{static}}}$$

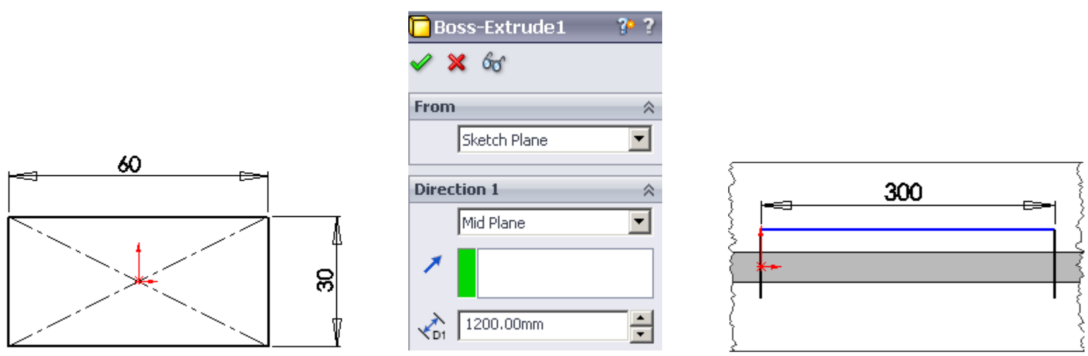
where *g* is the acceleration of gravity.

There are handbook equations for the static deflections of bars and beams that can be used to define the corresponding member stiffness, k. In general the static deflection is $\delta_{static} = W/k$, where k is the stiffness of the member, at the impact point, in the direction of the impact. For any object, a finite element static stress analysis can be probed to obtain the static deflection, in the direction of the force, at the point of loading to obtain the value of δ_{static}.

Here, the mechanics of materials solutions, finite element simulations, and a TK Solver worksheet will all be used to estimate the Impact Factor for example bars and beams. The first example (solved by all three methods) is for a weight dropped on the middle of a simply supported beam of rectangular cross-section. The mass of *80 kg* is dropped vertically *10 mm* from rest onto the center of a horizontal simply supported beam of length *1.2 m*. The beam is steel (*E = 200 GPa*) with a rectangular cross-section area having a width of *60 mm* and a height of *30 mm*. The first example causes an Impact Factor of about *6*. The second example will yield an Impact Factor above *1,200*.

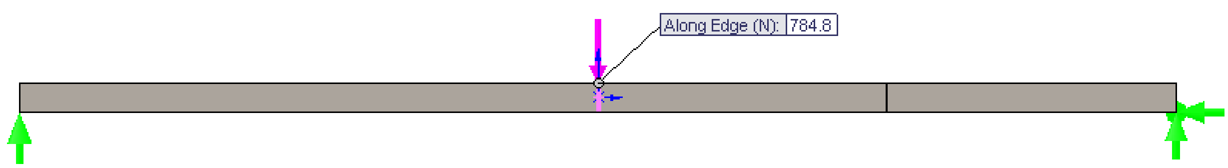
SolidWorks Finite Element Simulation:

The SolidWorks model begins the first example with the cross-section, extrudes it to the beam length, and adds vertical split lines to the solid to allow loading at the middle and/or quarter points, and to recover the deflection and stresses there.



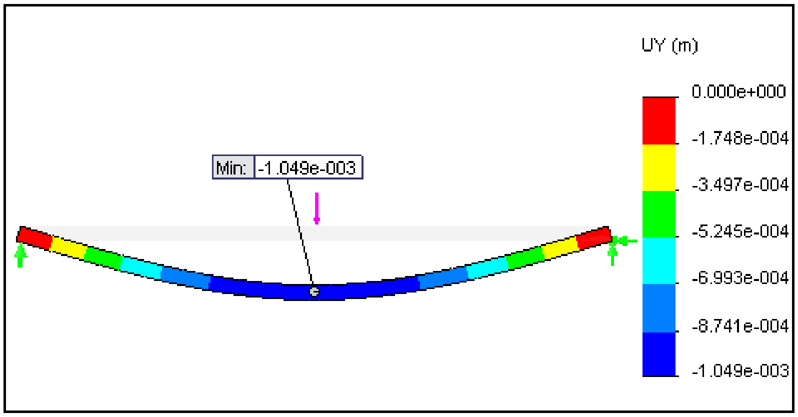
Generating the symmetric beam member, with split lines

The beam member is pinned at the right and supported with rollers on the left. The static weight (*W = 784.8 N*) is applied at the center point.



Static force applied vertically at beam center

After assigning the material properties, the static deflection at the point of loading is obtained:

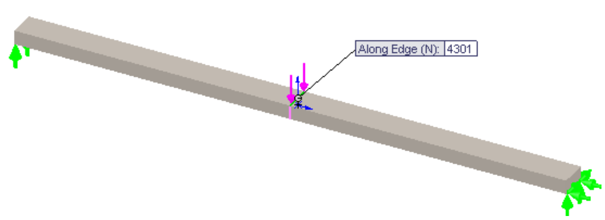


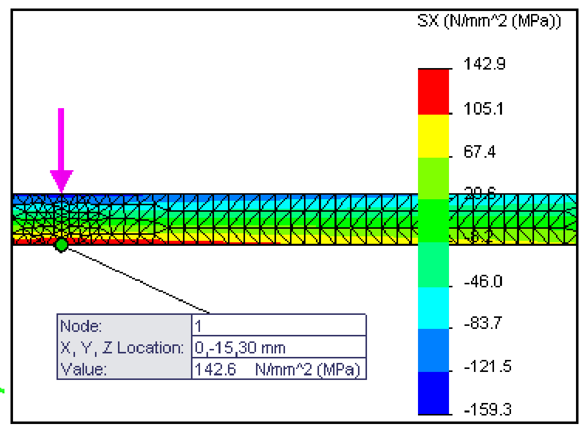
The static deflection at the load point

Its value, $\delta_{static} = 0.001049$ mm, is substituted into the above vertical factor equation, with 100 % efficiency, to yield

$$n = 1 + \sqrt{1 + \frac{2(0.010)1}{0.001049}} = 5.48$$

so the actual static force needs to be $P_{max} = n\,W \;=\; 5.48\,(784.8\,N) \;=\; 4{,}301\,N$. Applying that load gives a maximum dynamic horizontal fiber stress estimate of about *SX = 143 MPa*, as seen below.





Maximum horizontal stress (143 MPa) is at the center plane top and bottom

Of course, since the static model is linear there is no need to re-run the model (unless you want pretty plots). The static maximum fiber stress (SX) of 26.0 MPa could be multiplied by the 5.48 vertical Impact Factor to find the dynamic maximum SX stress of 143 MPa.

TK Solver Model

The TK Solver worksheet begins by writing a set of rules (from mechanics of solids and particle dynamics).

Status	Rule
Comment	; Static Load Amplification due to Impact Force
Comment	; Prof. J.E. Akin, Rice University, Sept. 2015
Satisfied	W = m * g ; weight applied (or mass)
Satisfied	2 * g * h = V ^2 ; impact speed from height h
Satisfied	F_static = W * δ_max / δ_s ; static load equivalent to W
Satisfied	W / k = δ_s ; static deflection for stiffness k
Satisfied	Factor_v = 1 + sqrt (1 + 2 * h * η / δ_s) ; vertical impact, with efficiency η
Satisfied	Factor_h = sqrt (V^2 * η / g / δ_s) ; horizontal impact, with efficiency η
Satisfied	δ_max = δ_s * Factor ; maximum dynamic deflection at pt
Comment	; NOTE δ_s can be found from a FEA study as the displacement component,
Comment	; in the force direction, at the applied load point
Comment	; Typical section geometry
Satisfied	A = B * H ; rectangle area, width B
Satisfied	I_r = B * H^3 / 12 ; rectangle inertia, depth H
Satisfied	A = pi() * r^2 ; area, circle of radius r
Satisfied	I_c = 0.25 * pi() * r^4 ; inertia, circle of radius r

The basic TK Solver rules for impact forces

Status	Rule
Comment	; Evaluate a few common stiffnesses to possibly copy into k
Satisfied	EA = E * A ; axial bar property
Satisfied	EI = E * I ; beam bending property
Comment	; Axial bar: fixed _________________W, L = total length
Satisfied	k_bar = EA / L ; vertical bar, W at end
Comment	; cantilever beam: fixed _________W__(b)___ free, L = total length (b not L)
* Unsatisfied	k_canti = 6 * EI / (2 * L^3 - 3 * L^2 * b + b^3) ; loaded distance b from free end
Comment	; simple beam: pin _____________W__(a)__ pin, L = total length (a not 0)
Satisfied	k_simple = 3 * EI * L / (a^2 * (L - a)^2) ; loaded distance a from support
Comment	; overhang simple: pin___________pin__(a) _W, L = total length (a not 0)
Satisfied	k_over = 3 * EI / (a^2 * L) ; overhanging, dist a beyond pin
Comment	; two bars in series: fixed ------- k_1 ------*----- k_2 ----- W
* Unsatisfied	1/k_1 + 1/k_2 = 1/k_series ; net stiffness

TK rules for five common member structural stiffnesses

The initial input variables (below) at the first solution (the first click on the TK lightbulb icon) were incomplete since neither the static deflection nor member stiffness was known. Thus, several rules were not satisfied as indicated by the lack of values in the Output column.

Input	Name	Output	Unit	Comment
				Equivalent Static Load for Impacting Force
				Prof. J.E. Akin, Rice University
1	η			efficiency of impact, 0 <= η <= 1, say 0.9
80	m		kg	mass of weight
	W	784.8	N	suddenly applied force W
.01	h		m	height dropped, or
	V	.4429447	m/sec	velocity at impact
	k		N/m	structural stiffness at load position
	δ_s		m	static deflection due to force W
	Factor_h			for horizontal impact
	Factor_v			for vertical impact
	Factor			Impact Factor, >=2
	F_static		N	equivalent load for static analysis
	δ_max		m	max dynamic deflection
200000	E		MPa	elastic modulus
1.2	L		m	bar or beam total length
	A	.0018	m^2	area of cross-section
	I	1.35E-7	m^4	moment of inertia of cross-section

Initial velocity results, without the beam stiffness

However, the stiffness of a simply supported beam at the center (load) point was calculated in the second solve (Lightbulb icon pick) from additional rules by inputting the beam properties, length, and load location (centered *0.6 m* from the support).

Input	Name	Output	Unit	Comment
				Typical stiffness values at load point
	k_bar	3E8	N/m	bar axial stiffness
	b		m	W location from cantilever tip
	k_canti		N/m	bending stiffness, if member is cantilever
.6	a		m	location of W on simple beam or overhang
	k_over	187500	N/m	bending stiffness, simple, overhang load
	k_series		N/m	effective bar series stiffness
	k_simple	750000	N/m	bending stiffness, for a simple beam
				Geometry related to cross-section
.06	B		m	width of rectangular area
.03	H		m	height of rectangular area
	r	.0239365	m	radius, if cross-section is circular
	EA	3.6E8	N	axial property
	EI	27000	N_m^2	bending property
				Miscellaneous items
9.81	g		m/sec^2	gravitational acceleration
	k_1		N/m	first bar in series
	k_2		N/m	second bar in series

Additional output of the beam stiffness for input for the member stiffness

There were enough inputs provided to calculate the stiffness of three of five typical members including the simply supported beam specified in this example (as seen above). Since the weight is dropped on the center of a simple beam, the value of that beam stiffness (*k = 7.5e5 N/m*) was copied from the output column, of k_simple, and pasted into the input column of *k*. Then, the next solve gives the desired Impact Factor values for both a horizontal or vertical impact.

Input	Name	Output	Unit	Comment
				Equivalent Static Load for Impacting Force
				Prof. J.E. Akin, Rice University
1	η			efficiency of impact, 0 <= η <= 1, say 0.9
80	m		kg	mass of weight
	W	784.8	N	suddenly applied force W
.01	h		m	height dropped, or
	V	.4429447	m/sec	velocity at impact
750000	k		N/m	structural stiffness at load position
	δ_s	.0010464	m	static deflection due to force W
	Factor_h	4.371859		for horizontal impact
	Factor_v	5.484769		for vertical impact
	Factor			Impact Factor, >=2
	F_static		N	equivalent load for static analysis
	δ_max		m	max dynamic deflection

Inputting the member stiffness at the load point gives the impact factors

Now the static deflection ($\delta_{static} = 1.05e\text{-}3\ m$) matches the finite element study well, as does the vertical impact factor of about 5.5. (Had the weight been held at the beam surface without toughing it (*h = 0*) and then released that calculation gives the vertical Impact Factor equal to two, and a zero horizontal value.) In this case, the vertical value was input as the impact factor and the next solve gave the equivalent force value as $P_{max} = 4{,}304\ N$, and the dynamic deflection was estimated as about $\delta_{max} = 6\ mm$.

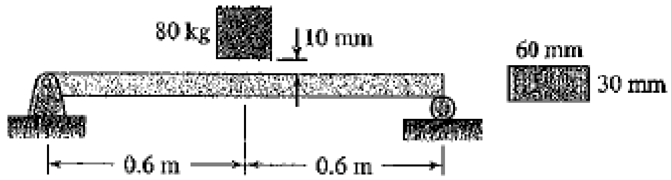
Input	Name	Output	Unit	Comment
	Factor_h	4.371859		for horizontal impact
	Factor_v	5.484769		for vertical impact
5.484769	Factor			Impact Factor, >=2
	F_static	4304.446	N	equivalent load for static analysis
	δ_max	.0057393	m	max dynamic deflection

The final solve gives the equivalent static force (about 4,300 N)

The following figure is an image of a hand solution example taken from the "Mechanics of Materials" text by A. Pytel and J. Kiusalaas, Thomson, 2003. Those results match the finite element solution, and the TK Solver solutions, as expected.

Sample Problem **12.5** Pytel, Kiusalaas "Mech of Materials"

The 80-kg block hits the simply supported beam at its midspan after a drop of 10 mm as shown in Fig. (a). Determine (1) the impact factor; and (2) the maximum dynamic bending stress in the beam. Use $E = 200$ GPa for the beam. Assume that the block and the beam stay in contact after the collision.



(a)

Solution

Part 1

The moment of inertia of the cross section of the beam abcut the neutral axis is

$$I = \frac{bh^3}{12} = \frac{60(30)^3}{12} = 135.0 \times 10^3 \text{ mm}^4 = 135.0 \times 10^{-9} \text{ m}^4$$

According to Table 6.3 on page 233, the static midspan deflection of the beam under the weight of the 80-kg mass is

$$\delta_{st} = \frac{(mg)L^3}{48EI} = \frac{(80 \times 9.81)(1.2)^3}{48(200 \times 10^9)(135.0 \times 10^{-9})} = 1.0464 \times 10^{-3} \text{ m}$$

From Eq. (12.12b), the impact factor is

$$n = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} = 1 + \sqrt{1 + \frac{2(0.010)}{1.0464 \times 10^{-3}}} = 5.485 \qquad \textit{Answer}$$

Part 2

The maximum dynamic load P_{max} at the midspan of the beam is obtained by multiplying the static load by the impact factor:

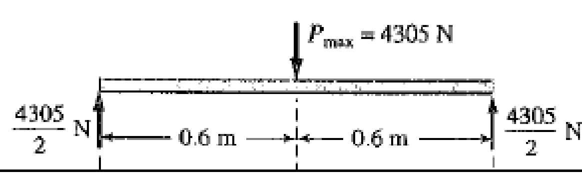
$$P_{max} = n(mg) = 5.485(80 \times 9.81) = 4305 \text{ N}$$

The maximum bending moment caused by this load occurs at the midspan, as shown in Fig. (b). Its value is

$$M_{max} = \frac{4305}{2}(0.6) = 1291.5 \text{ N} \cdot \text{m}$$

which results in the maximum dynamic bending stress

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{1291.5(0.015)}{135.0 \times 10^{-9}} = 143.5 \times 10^6 \text{ Pa} = 143.5 \text{ MPa} \qquad \textit{Answer}$$



The second example models a *1 kg* mass moving horizontally along a steel (*E = 207 GPa*) bar *100 mm* long with a *10 mm* diameter and striking the end of the bar with a velocity of *1 m/sec*. (It is taken from R.L. Norton, Machine Design, Prentice Hall, 1996.) It is assumed that the correction for the mass of the struck bar reduces the efficiency to $\eta = 0.98$. These values were input in the first solve, but the impact factor cannot be obtained without a member stiffness. So the first solution was to actually find the stiffness of the axial bar, so it could be copied to the member stiffness input.

Input	Name	Output	Unit	Comment
				Equivalent Static Load for Impacting Force
				Prof. J.E. Akin, Rice University
				(Norton example)
.98	η			efficiency of impact, 0 <= η <= 1, say 0.9
1	m		kg	mass of weight
	W	9.81	N	suddenly applied force W
	h	50.9684	mm	height dropped, or
1	V		m/s	velocity at impact
	k		N/m	structural stiffness at load position
	δ_s		mm	static deflection due to force W
	Factor_h			for horizontal impact
	Factor_v			for vertical impact
	Factor			Impact Factor, >=2
	F_static		N	equivalent load for static analysis
	δ_max		mm	max dynamic deflection
207	E		GPa	elastic modulus
100	L		mm	bar or beam total length
	A	78.53982	mm^2	area of cross-section
	I		mm^4	moment of inertia of cross-section
				Typical stiffness values at load point
	k_bar	1.6258E8	N/m	bar axial stiffness
	b		m	W location from cantilever tip
	k_canti		N/m	bending stiffness, if member is cantilever
	a		mm	location of W on simple beam or overhang
	k_over		N/m	bending stiffness, simple, overhang load
	k_series		N/m	effective bar series stiffness
	k_simple		N/m	bending stiffness, for a simple beam
				Geometry related to cross-section
	B		mm	width of rectangular area
	H		mm	height of rectangular area
5	r		mm	radius, if cross-section is circular

First solve mainly finds the bar member stiffness to be copied

Having the member stiffness, the impact factors are obtained along with a value of the "static deflection". However, the latter (vertical load) value is only for reference since the impact is horizontal.

Input	Name	Output	Unit	Comment
.98	η			efficiency of impact, 0 <= η <= 1, say 0.9
1	m		kg	mass of weight
	W	9.81	N	suddenly applied force W
	h	50.9684	mm	height dropped, or
1	V		m/s	velocity at impact
1.6258E8	k		N/m	structural stiffness at load position
	δ_s	6.034E-5	mm	static deflection due to force W
	Factor_h	1286.691		for horizontal impact
	Factor_v	1287.691		for vertical impact
	Factor			Impact Factor, >=2
	F_static		N	equivalent load for static analysis
	δ_max		mm	max dynamic deflection

The member stiffness yields the horizontal factor to copy

Input	Name	Output	Unit	Comment
.98	η			efficiency of impact, 0 <= η <= 1, say 0.9
1	m		kg	mass of weight
	W	9.81	N	suddenly applied force W
	h	50.9684	mm	height dropped, or
1	V		m/s	velocity at impact
1.6258E8	k		N/m	structural stiffness at load position
	δ_s	6.034E-5	mm	static deflection due to force W
	Factor_h	1286.691		for horizontal impact
	Factor_v	1287.691		for vertical impact
1286.691	Factor			Impact Factor, >=2
	F_static	12622.44	N	equivalent load for static analysis
	δ_max	.0776395	mm	max dynamic deflection

An impact factor defines the equivalent static force and the dynamic displacement

This is a very large impact factor. However, the dynamic deflection still leads to a small elastic strain: $\varepsilon = \delta/L = 0.08\ mm/100\ mm < 0.1$. Thus, energy loss due to plastic action does not have to be estimated. However, there are corrections published to account for the mass of the bar. Typical additional rules to correct the efficiency for the mass of the member are listed below. It is well known that for bars of a constant diameter the Impact Factor decreases rapidly with increased bar length.

The equations for the Impact Factors neglect the mass of the member being struck. There are handbook equations that reduce the efficiency to less than unity by including the mass of the struck member. Some of those corrects are also provided as rules, but they rarely reduce the efficiency below 0.95. In this case, the bar mass correction only reduces efficiency to $\eta = 0.96$

as shown in the figures below.

Status	Rule
Comment	; Efficiency estimates, special cases (Ref: Roark's Formulas)
Satisfied	η_bar = bar_n / bar_d ; impact at end of bar
Satisfied	bar_n = 1 + m_member / m / 3
Satisfied	bar_d = (1 + m_member / m / 2)^2
Satisfied	η_simple = simple_n / simple_d ; impact at center a = L/2
Satisfied	simple_n = 1 + 17 * m_member / m / 35
Satisfied	simple_d = (1 + 5 * m_member / m / 8)^2
Satisfied	η_canti = canti_n / canti_d ; at cantilever tip b = 0
Satisfied	canti_n = 1 + 33 * m_member / m / 140
Satisfied	canti_d = (1 + 3 * m_member / m / 8)^2
Satisfied	m_member = ρ * A * L ; mass of member

Correction rules for the member mass

Input	Name	Output	Unit	Comment
7.86	ρ		g/cc	mass density of member
	m_member	.0617323	kg	mass of member
	η_bar	.9603762		end efficiency
	η_simple	.9548792		middle efficiency
	η_canti	.9691604		end efficiency

Efficiency correction due to member mass

Input	Name	Output	Unit	Comment
.9603762	η			efficiency of impact, 0 <= η <= 1, say 0.9
1	m		kg	mass of weight
	W	9.81	N	suddenly applied force W
	h	50.9684	mm	height dropped, or
1	V		m/s	velocity at impact
1.6258E8	k		N/m	structural stiffness at load position
	δ_s	6.034E-5	mm	static deflection due to force W
	Factor_h	1273.743		for horizontal impact
	Factor_v	1274.743		for vertical impact
1273.743	Factor			Impact Factor, >=2
	F_static	12495.42	N	equivalent load for static analysis
	δ_max	.0768583	mm	max dynamic deflection

The member mass correction has little effect on the maximum dynamic displacement

For a third example the effects of dropping an *80 kg* mass from different heights onto the tip of a cantilever beam of 1,200 mm length will be illustrated. The stiffness of the cantilever is *46,875 N/m*. Using that member stiffness and drop heights of 0.01, 0.10, and 1 meter yields vertical impact factors of 2.48, 4.60, and 12.0, respectively as seen in the following outputs.

Input	Name	Output	Unit	Comment
	W	784.8	N	suddenly applied force W
.01	h		m	height dropped, or
	V	.4429447	m/sec	velocity at impact
46875	k		N/m	structural stiffness at load position
	δ_s	.0167424	m	static deflection due to force W
	Factor_h	1.092965		for horizontal impact
	Factor_v	2.481409		for vertical impact

Input	Name	Output	Unit	Comment
	W	784.8	N	suddenly applied force W
.1	h		m	height dropped, or
	V	1.400714	m/sec	velocity at impact
46875	k		N/m	structural stiffness at load position
	δ_s	.0167424	m	static deflection due to force W
	Factor_h	3.456258		for horizontal impact
	Factor_v	4.598016		for vertical impact

Input	Name	Output	Unit	Comment
	W	784.8	N	suddenly applied force W
1	h		m	height dropped, or
	V	4.429447	m/sec	velocity at impact
46875	k		N/m	structural stiffness at load position
	δ_s	.0167424	m	static deflection due to force W
	Factor_h	10.92965		for horizontal impact
	Factor_v	11.9753		for vertical impact

Cantilever beam Impact Factor values for drop heights of 0.01, 0.10, and 1.00 meters

As a final example, R.C. Hibbeler, Mechanics of Materials, Prentice Hall, 1999 gives the following impact problem. An *80 Mg* railway car moves horizontally with a speed of *0.2 m/sec* when it impacts a tract end post. The steel cantilever post is *200 mm* by *200 mm* in cross-section and the length of the post, at the point of contract, is *1.5 m*. Determine the maximum dynamic displacement of the post tip and the equivalent static force for determining the post stresses.

The first solve is just used to find the bending inertia of the post for two common shapes.

Input	Name	Output	Unit	Comment
200	B		mm	width of rectangular area
200	H		mm	height of rectangular area
	r	112.8379	mm	radius, if cross-section is circular
	I_c	.0001273	m^4	inertia if circular
	I_r	.0001333	m^4	inertia if rectangular
	EA	8E9	N	axial property
	EI	26666667	N_m^2	bending property

Cross-section inertia values

The rectangular inertia value is copied into the beam member inertia so that typical beam and bar stiffnesses are evaluated, in the second solve. The cantilever stiffness value is copied into the member stiffness input so the third solve yields the horizontal impact factor of about 0.35.

Input	Name	Output	Unit	Comment
200	E		GPa	elastic modulus
1.5	L		m	bar or beam total length
	A	40000	mm^2	area of cross-section
.0001333	I		m^4	moment of inertia of cross-section
				Typical stiffness values at load point
	k_bar	30454263	lb/in	bar axial stiffness
0	b		m	W location from cantilever tip
	k_canti	23703704	N/m	bending stiffness, if member is cantilever

Input a member inertia and obtain stiffness for a cantilever beam

Input	Name	Output	Unit	Comment
				Equivalent Static Load for Impacting Force
				Prof. J.E. Akin, Rice University
				(Hibbeler example)
1	η			efficiency of impact, 0 <= η <= 1, say 0.98
80000	m		kg	mass of weight
	W	784800	N	suddenly applied force W
	h	2.038736	mm	height dropped, or
.2	V		m/s	velocity at impact
23703704	k		N/m	structural stiffness at load position
	δ_s	1.303494	in	static deflection due to force W
	Factor_h	.3509329		for horizontal impact
	Factor_v	2.05979		for vertical impact
	Factor			Impact Factor, >=2
	F_static		kN	equivalent load for static analysis
	δ_max		mm	max dynamic deflection

The member stiffness yields the horizontal impact factor to be copied

The final solve gives the dynamic displacement as about $\delta_{\max} = 11.6\text{ mm}$ and the equivalent static force of about $P_{\max} = 275\text{ kN}$, as seen below. There are enough data to obtain the correction factor for the post mass but it gives (below) almost complete efficiency, so there is no need to re-run this set of calculations.

Input	Name	Output	Unit	Comment
				Equivalent Static Load for Impacting Force
				Prof. J.E. Akin, Rice University
				(Hibbeler example)
1	η			efficiency of impact, 0 <= η <= 1, say 0.98
80000	m		kg	mass of weight
	W	784.8	kN	suddenly applied force W
	h	2.038736	mm	height dropped, or
.2	V		m/s	velocity at impact
23703704	k		N/m	structural stiffness at load position
	δ_s	33.10875	mm	static deflection due to force W
	Factor_h	.3509329		for horizontal impact
	Factor_v	2.05979		for vertical impact
.3509329	Factor			Impact Factor, >=2
	F_static	275.4121	kN	equivalent load for static analysis
	δ_max	11.61895	mm	max dynamic deflection

The final horizontal equivalent static force

Input	Name	Output	Unit	Comment
				Estimates of efficiency
7.86	ρ		g/cc	mass density of member
	m_member	471.6	kg	mass of member
	η_bar	.9960844		end efficiency
	η_simple	.9955141		middle efficiency
	η_canti	.9969768		end efficiency

A correction for the member mass is not required