17 Related Analogies

17.1 Basic Concepts

The differential equation used in a finite element study in one discipline often appears in a different discipline, but with a different physical meaning for the unknown and the coefficients in the equation. That is particularly true for the diffusion equation (heat transfer here) and the biharmonic equation (flat plate deflection here). They are the most common second order and fourth order differential equations in engineering. Consider the slightly generalized 2D field equation, in the solution domain:

$$k_x \frac{\partial^2 \varphi}{\partial x^2} + k_y \frac{\partial^2 \varphi}{\partial y^2} + P = 0$$

Subject to a Dirichlet boundary condition on boundary segment Γ_D of

$$\varphi = \varphi_{given}$$

or a Neumann boundary condition (known normal flux) on boundary segment Γ_N of

$$k_n \frac{\partial \varphi}{\partial n} = f_{given}$$

or a Convection (Robin) boundary condition on boundary segment Γ_R of

$$k_n \frac{\partial \varphi}{\partial n} = h(\varphi - \varphi_\infty) = h\varphi + g$$

where the boundaries do not overlap. The meanings of the above symbols, in a few disciplines, are listed in Table 17-1. These analogies allow you to use SW Simulation to solve problems in such fields by replacing the SW Simulation inputs with corresponding values (and units) for the field of interest. You should also edit the graphic outputs to show the desired terminology (as done here with the SnagIt software).

Table 17-1 Some general field equation terms

Field	φ	k	P	h	φ∞	f
Heat Transfer	Temperature	Thermal Conductivity	Heat Power	Convection Coefficient	Convection Temperature	Boundary Flux
Electric Conduction	Voltage	Electrical Conductivity	Current Source	0	0	Boundary Current
Porous Media Flow	Hydraulic Head	Hydraulic Conductivity	Flow Source	0	0	Boundary Flow
Irrotational Flow	Velocity Potential	1	Flow Source	0	0	Boundary Velocity

17.2 Seepage under a dam

Water in a reservoir will almost always seep under and/or around a dam. It needs to be controlled to have a low velocity so that the region around the dam will not be eroded away. This seepage, or porous media flow, field will be illustrated for a dam resting on layered, and thus orthotropic, soils [13]. The dimensions of the soil regions, dam, and toe wall are given in Figure 17-1. The left side of the dam holds water 30 m deep, while the right side is 1 m deep. Split lines locate the impervious dam interface at the soil top. The far boundaries of the soil are also impervious (no normal flow, $f = 0$, the natural). The layer soil permeabilities (or hydraulic conductivities) are *20 m/day* and *15 m/*day in the horizontal and vertical directions, respectively. Those two orthotropic properties are specified with respect to the Front Plane and input as seen in Figure 17-2. Usually layered soils are inclined and require the use of a reference plane to define the principal material directions.

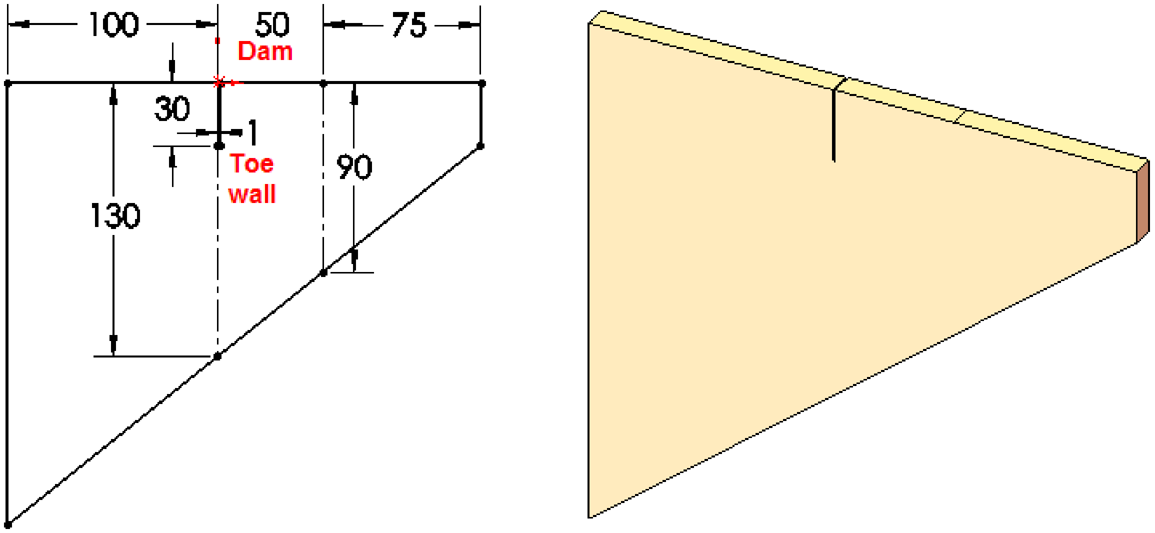
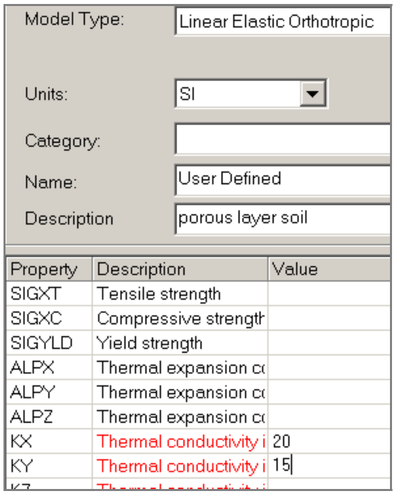


Figure 17-1 Orthotropic soil beneath a dam



Model Type: Linear Elastic Orthotropic

Units: SI

Category:

Name: User Defined

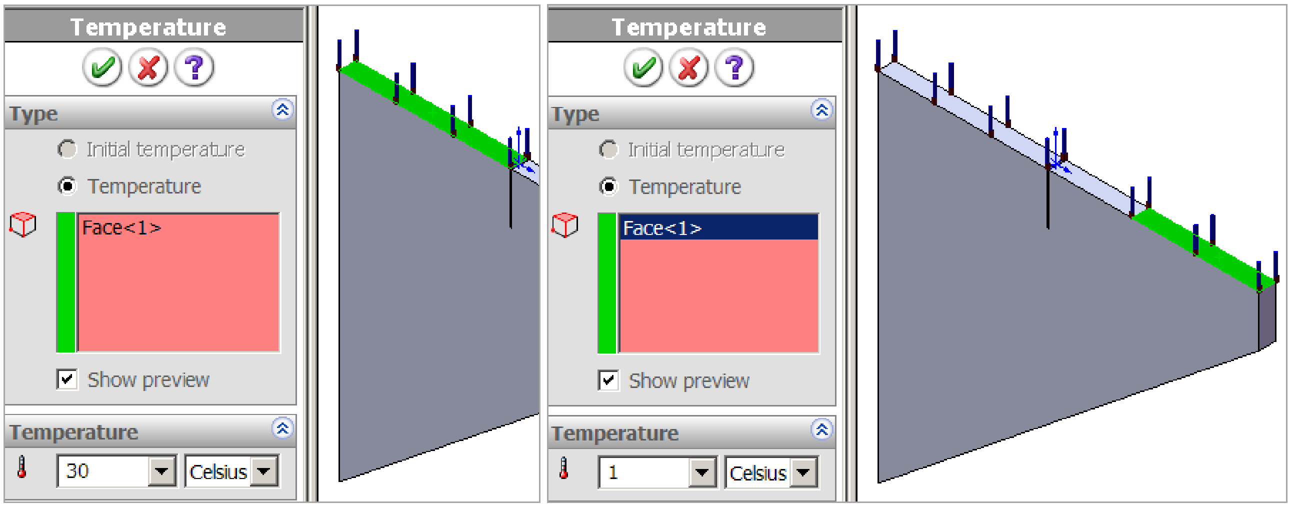
Description porous layer soil

Property	Description	Value
SIGXT	Tensile strength	
SIGXC	Compressive strengtr	
SIGYLD	Yield strength	
ALPX	Thermal expansion cc	
ALPY	Thermal expansion cc	
ALPZ	Thermal expansion cc	
KX	Thermal conductivity i	20
KY	Thermal conductivity i	15

Figure 17-2 Input soil permeabilities in the Front Plane

17.2.1.1 Restraints

The constant hydraulic head on either side of the dam are like specified temperatures. Of course, the chosen unit of *Celsius* here represents *m* (meters of water). Those two essential restraints are seen in Figure 17-3.



Temperature

Type

○ Initial temperature

● Temperature

Face<1>

☑ Show preview

Temperature

30 Celsius

Temperature

Type

○ Initial temperature

● Temperature

Face<1>

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Temperature

1 Celsius

Figure 17-3 Set water pressure boundary values

17.2.1.2 *Mesh and execute*

The very narrow toe wall constructed at the front of the dam causes a sharp reentrant corner in the soil (almost a crack). That means very high gradients (velocities) will occur there. Therefore, it is necessary to invoke mesh control there to force small elements at the base of that wall. A portion of the soil mesh is given in Figure 17-4. Now you can **Run** the study. The resulting soil pressures are given in Figure 17-5.

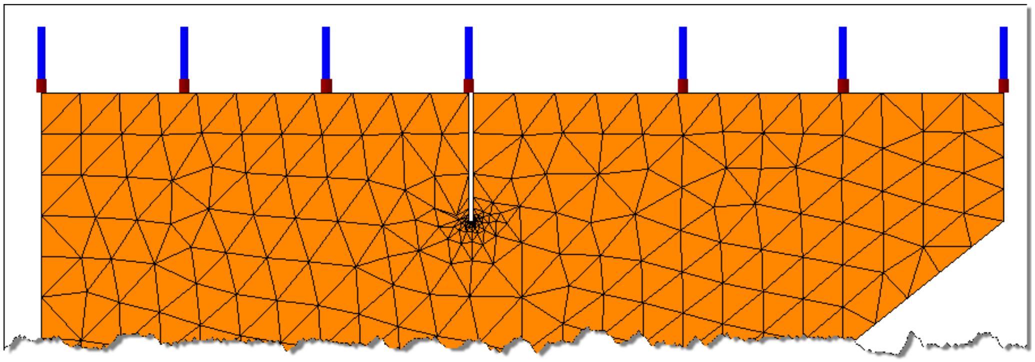


Figure 17-4 Refined mesh around the toe wall tip

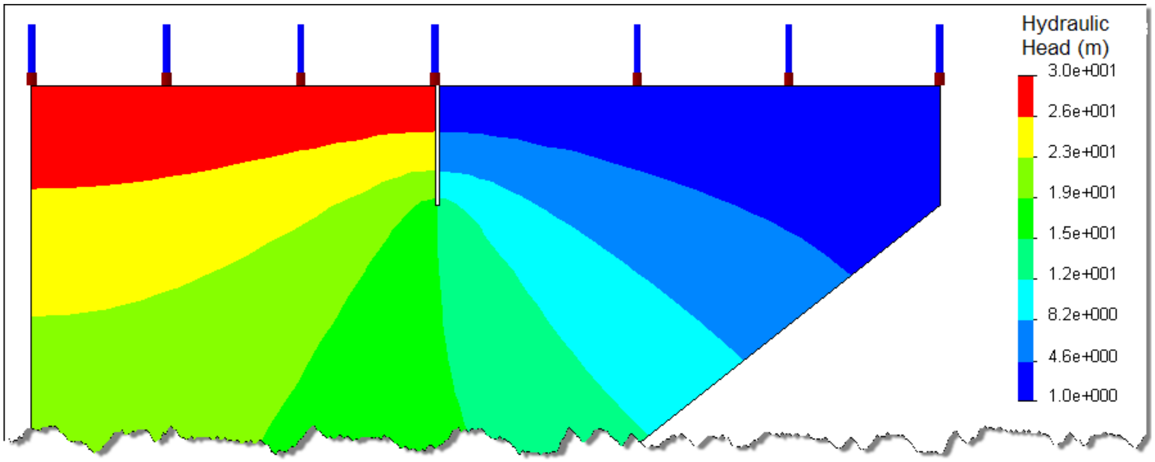


Figure 17-5 Hydraulic head (water pressure) in the soil, with edited color bar

17.2.1.3 Seepage velocity vectors

To see the velocity vectors, just select the SW Simulation heat flux vector plot and re-label the color bar to display the units of m/day (with SnagIt, etc.). As desired, the velocities are quite small through the soil. The largest vales occur where the water changes directions from down to up around the toe wall tip (see Figure 17-6).

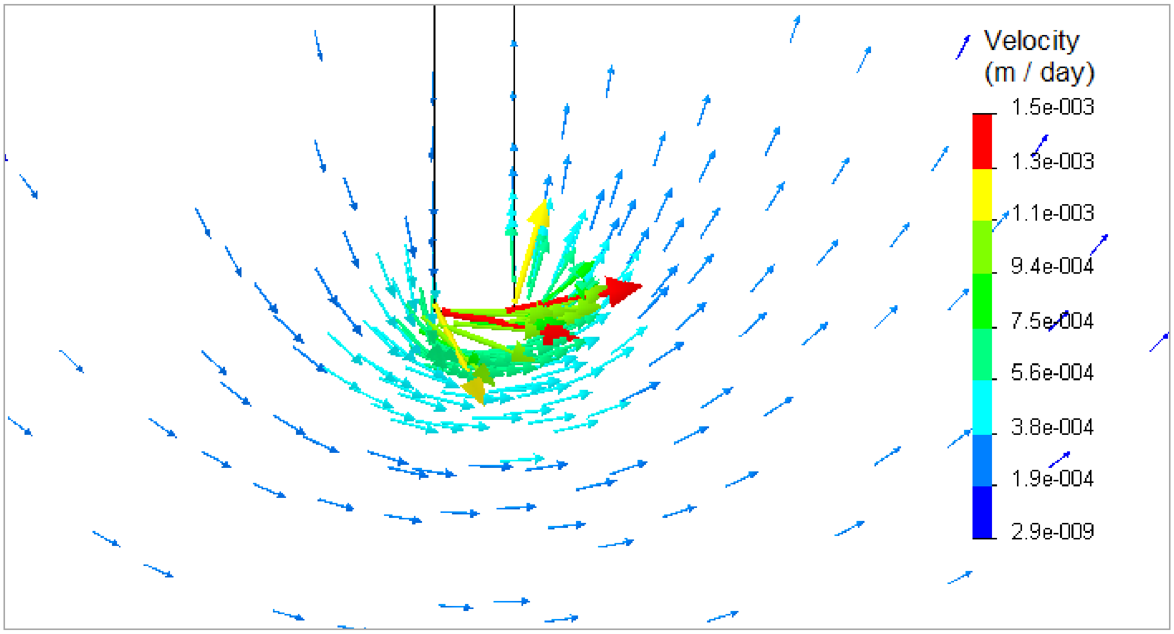
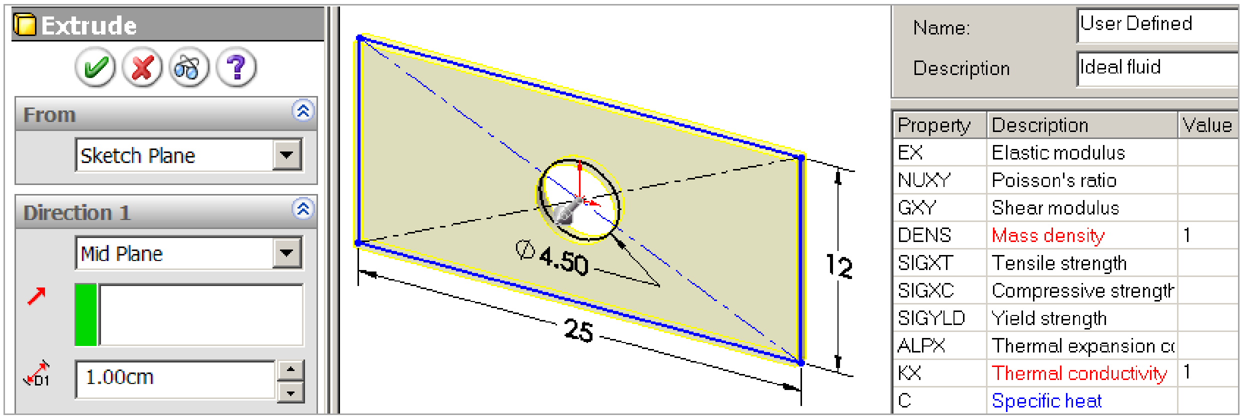


Figure 17-6 Seepage velocities at the toe wall tip

17.3 Potential flow around a cylinder

Consider the Irrotational flow of an ideal fluid around solid cylinder within a rectangular channel dimensioned as shown in Figure 17-7. Its properties are unity as given in Figure 17-1. The fluid enters at the left with a constant normal velocity of 5 cm/sec, and exits at the right with the same speed in order to conserve mass. That means that only Neumann boundary conditions are required in theory to determine the value of the velocity potential to within an arbitrary constant. In practice, due to machine accuracy slightly violating mass conservation, you should pick one point to assign an arbitrary value to the potential. Here, use the top wall point centered over the cylinder (Figure 17-8).



From: Sketch Plane

Direction 1: Mid Plane

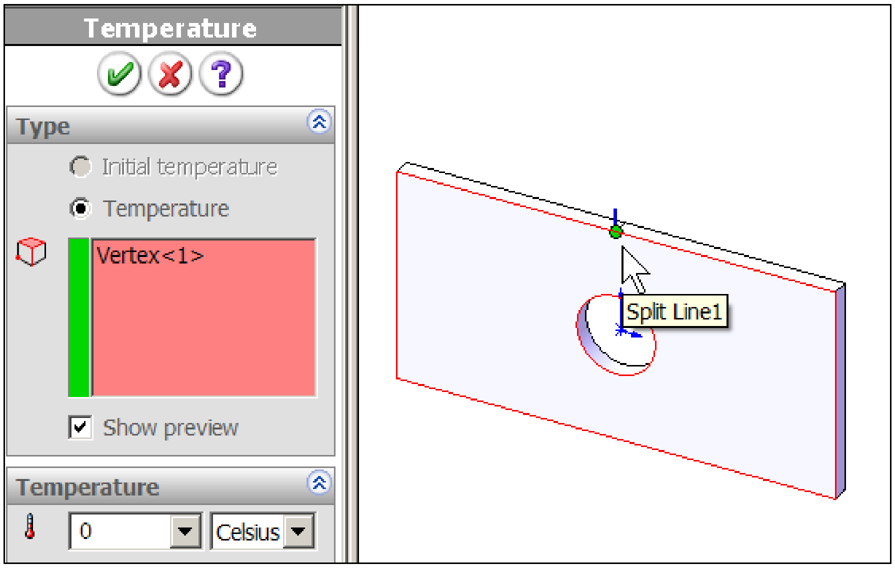
1.00cm

Name: User Defined

Description: Ideal fluid

Property	Description	Value
EX	Elastic modulus	
NUXY	Poisson's ratio	
GXY	Shear modulus	
DENS	Mass density	1
SIGXT	Tensile strength	
SIGXC	Compressive strength	
SIGYLD	Yield strength	
ALPX	Thermal expansion co	
KX	Thermal conductivity	1
C	Specific heat	

Figure 17-7 Fluid around a solid cylinder in a rectangular channel



Type

○ Initial temperature

● Temperature

Vertex<1>

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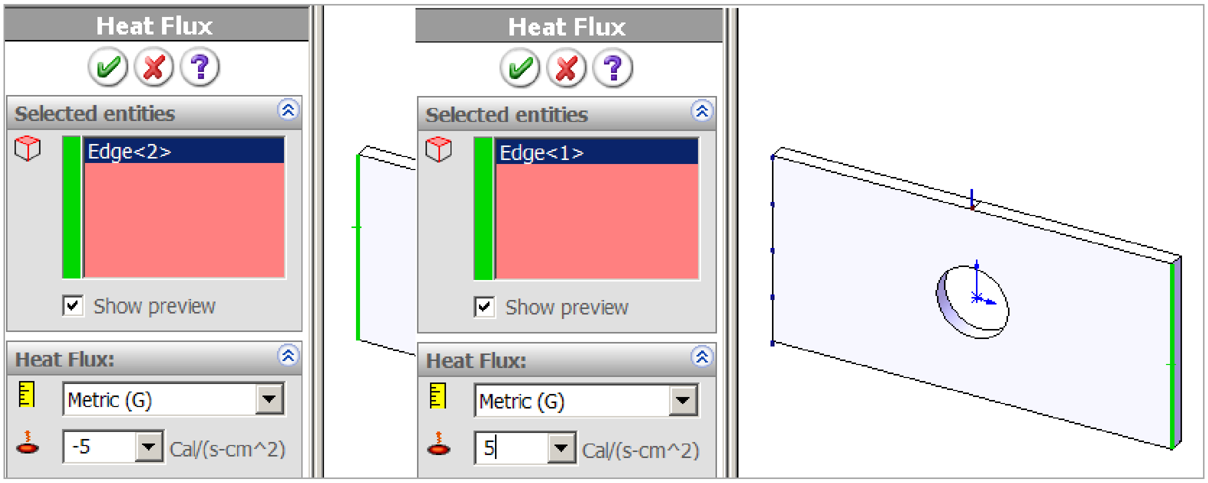
Temperature: 0 Celsius

Figure 17-8 Specify the potential at an arbitrary point

17.3.1 Inflow and outflow boundary sources

For potential flow, a velocity inward across a boundary is negative. The sources must satisfy mass conservation. Since the length of the outlet is the same as the inlet only the sign changes at the right end outflow. Those two flow loads are illustrated in Figure 17-9. The mesh, in Figure 17-10, was controlled to be finer where the velocities are expected to change rapidly around the cylinder.



Heat Flux

Selected entities: Edge<2>

☑ Show preview

Heat Flux: Metric (G)

-5 Cal/(s-cm^2)

Heat Flux

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Heat Flux: Metric (G)

5 Cal/(s-cm^2)

Figure 17-9 Inflow and outflow boundary restraint (cm/sec)

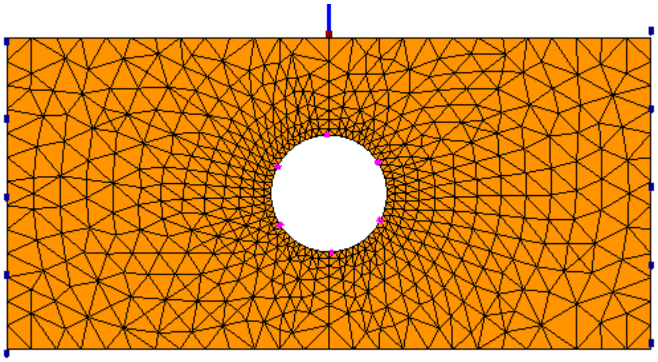


Figure 17-10 Graded mesh around the cylinder

17.3.2 Analysis results

The primary unknown, velocity potential, does not have a physical meaning but its value shown in Figure 17-11 confirms the expected anti-symmetric distribution. The velocity magnitudes and vectors are given in Figure 17-12 and Figure 17-13, respectively.

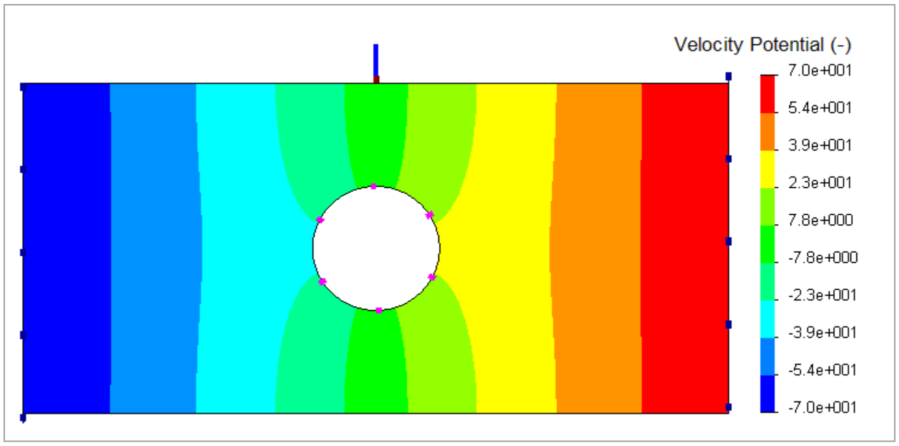


Figure 17-11 Anti-symmetric velocity potential around the cylinder

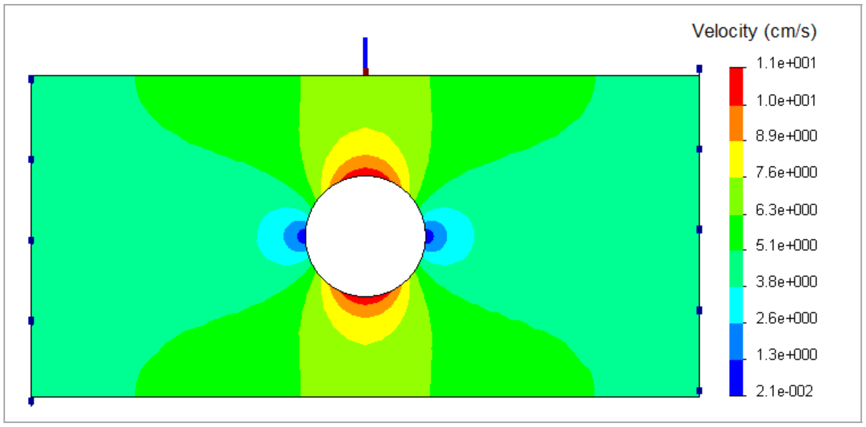


Figure 17-12 Fluid speed with anti-symmetric boundary flows

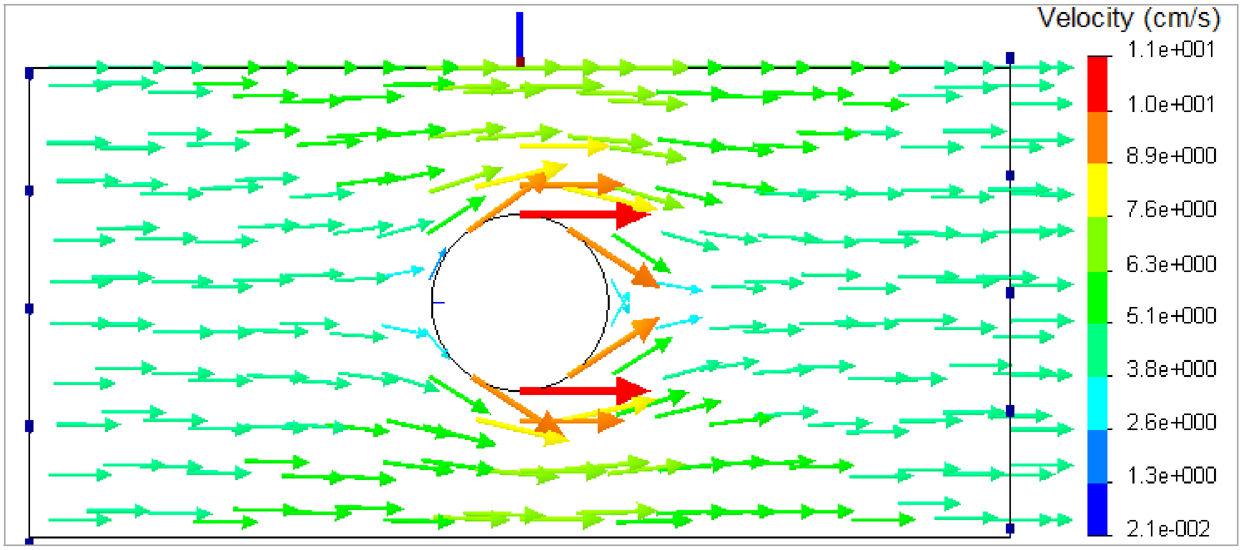
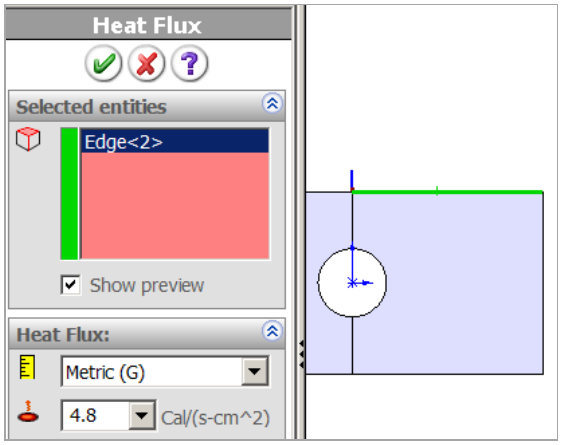


Figure 17-13 Velocity vectors with anti-symmetric boundary flows

17.3.3 Alternate outflow region

A considerably different result is obtained for other outlets. Here, half of the top channel edge is utilized. To conserve mass the normal flow component must be reduced, in Figure 17-14. That produces the new velocity vectors of Figure 17-15.



Heat Flux

Selected entities

Edge<2>

☑ Show preview

Heat Flux:

Metric (G)

4.8 Cal/(s-cm^2)

Figure 17-14 Modify the normal outflow direction and value, cm/sec

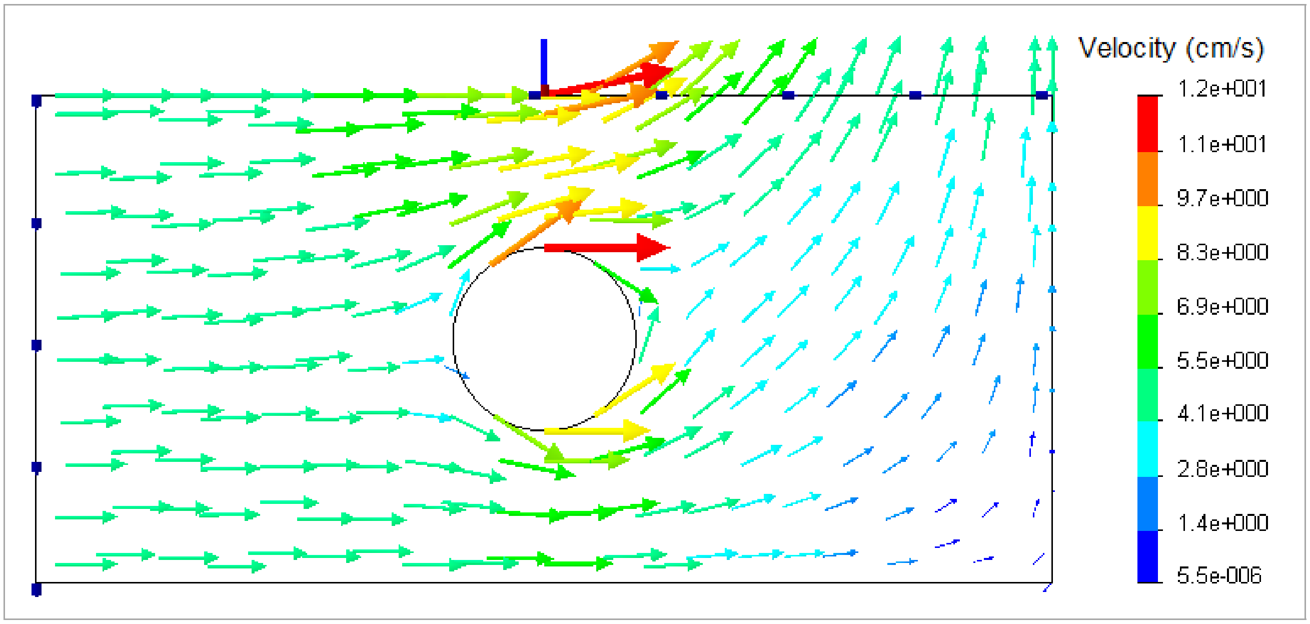


Figure 17-15 Velocity vectors for side outflow

17.4 Closure

Be alert for analogies that can extend the power and usefulness of you finite element software. Many commercial systems offer specific input and output interfaces for the alternate disciplines, but the underlying numerical calculations are basically the same. Minor exceptions are the torsional analogy and the pressurized membrane analogy which both utilize the integral of the solution additional as partial output.