# Concepts of Buckling

Draft 1, 10/23/06

## *Introduction*

There are two major categories leading to the failure of a mechanical component: material failure and structural instability, which is often called buckling. For material failures you need to consider the yield stress for ductile materials and the ultimate stress for brittle materials. Those material properties are determined by axial tension tests and axial compression tests of short columns of the material (see Figure 1). The geometry of such test specimens has been standardized. Thus, geometry is not specifically addressed in defining material properties, such as yield stress. Geometry enters the problem of determining material failure only indirectly as the stresses are calculated by analytic or numerical methods.

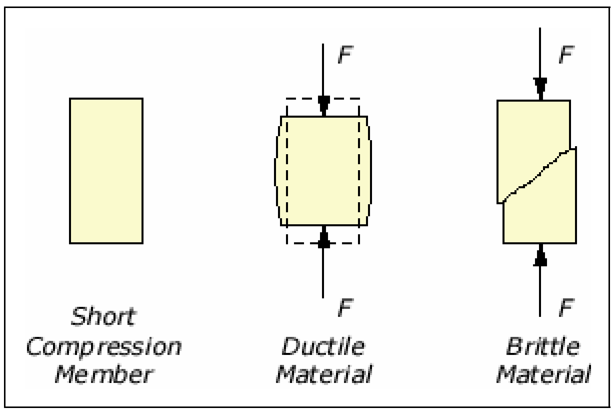


**Figure 1**

Predicting material failure may be accomplished using linear finite element analysis. That is, by solving a linear algebraic system for the unknown displacements, $\mathbf{K}\,\boldsymbol{\delta} = \mathbf{F}$. The strains and corresponding stresses obtained from this analysis are compared to design stress (or strain) allowables everywhere within the component. If the finite element solution indicates regions where these allowables are exceeded, it is assumed that material failure has occurred.

The load at which buckling occurs depends on the stiffness of a component, not upon the strength of its materials. Buckling refers to the loss of stability of a component and is usually independent of material strength. This loss of stability usually occurs within the elastic range of the material. The two phenomenon are governed by different differential equations. Buckling failure is primarily characterized by a loss of structural stiffness and is not modeled by the usual linear finite element analysis, but by a finite element eigenvalue-eigenvector solution, $|\mathbf{K} + \lambda_m \mathbf{K_F}|\, \delta_m = 0$, where $\lambda_m$ is the buckling load factor (BLF) for the m-th mode, $\mathbf{K_F}$ is the additional "geometric stiffness" due to the stresses caused by the loading, **F**, and $\delta_m$ is the associated buckling displacement shape for the m-th mode. The spatial distribution of the load is important, but its relative magnitude is not. The buckling calculation gives a multiplier that scales the magnitude to that required to cause buckling.

Slender or thin-walled components under compressive stress are susceptible to buckling. Most people have observed what is called "Euler buckling" where a long slender member subject to a compressive force moves lateral to the direction of that force, as illustrated in Figure 2. The force, F, necessary to cause such a buckling motion will vary by a factor of four depending only on how the two ends are restrained. Therefore, buckling studies are much more sensitive to the component restraints that in a normal stress analysis. The theoretical Euler solution will lead to infinite forces in very short columns, and that clearly exceeds the allowed material stress. Thus in practice, Euler column buckling can only be applied in certain regions and empirical transition equations are required for intermediate length columns. For very long columns the loss of stiffness occurs at stresses far below the material failure.

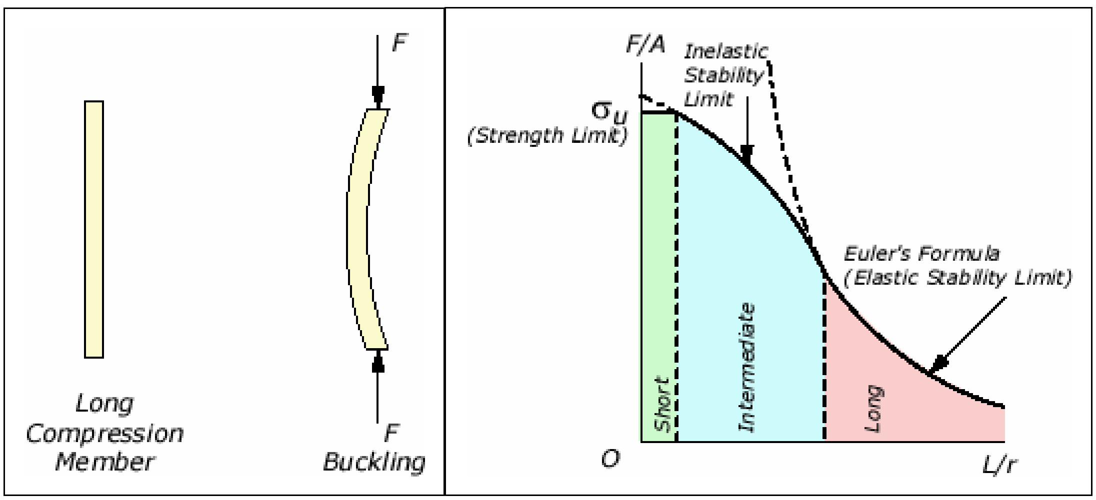


**Figure 2**

There are many analytic solutions for idealized components having elastic instability. About 75 of the most common cases are tabulated in the classic reference "Roark's Formulas for Stress and Strain" [1, 2, 3].

## *Buckling terminology*

The topic of buckling is still unclear because the keywords of "stiffness", "long" and "slender" have not been quantified. Most of those concepts were developed historically from 1D studies. You need to understand those terms even though finite element analysis lets you conduct buckling studies in 1D, 2D, and 3D. For a material, stiffness refers to either its elastic modulus, E, or to its shear modulus, $G = E / (2 + 2v)$ where $v$ is Poisson's ratio.

Slender is a geometric concept of a two-dimensional area that is quantified by the radius of gyration. The radius of gyration, r, has the units of length and describes the way in which the area of a cross-section is distributed around its centroidal axis. If the area is concentrated far from the centroidal axis it will have a greater value of r and a greater resistance to buckling.

 

A non-circular cross-section will have two values for its radius of gyration. The section tends to buckle around the axis with the smallest value. The radius of gyration is defined as:

$$r = \sqrt{(I / A)},$$

where r = radius of gyration, I = area moment of inertia, and A = area of the cross-section. For a circle of radius R, r = R / 2. For a rectangle of large length R and small length b you obtain $r_{max} = R / 2\sqrt{3} = 0.29$ R and $r_{min} = 0.29$ b. Solids can have regions that are slender, and if they carry compressive stresses a buckling study is justified.

Long is also a geometric concept that is quantified by the non-dimensional "slenderness ratio" L / r, where L denotes the length of the component. From experiments, the slenderness ratio of 120 is generally considered as the dividing point between long (Euler) columns (> 120) and intermediate (empirical) columns. The critical compressive stress that will cause buckling always decreases as the slenderness ratio increases.

Other 1D concepts that relate to stiffness are: axial stiffness, E A / L, flexural (bending) stiffness, E I / L, and torsional stiffness, G J / L, where J is the polar moment of inertia of the cross-sectional area ($J = I_z = I_x + I_y$). Today, stiffness usually refers to the finite element stiffness matrix, which can include all of the above stiffness terms plus general solid or shell stiffness contributions. Analytic buckling studies identify additional classes of instability besides Euler buckling. They include lateral buckling, torsional buckling, and other buckling modes (see Figure 3). A finite element buckling study determines the lowest buckling factors and their corresponding displacement modes. The amplitude of a buckling displacement mode, $|\boldsymbol{\delta}_m|$, is arbitrary and not useful, but the shape of the mode can suggest whether lateral, torsional, or other behavior is governing the buckling response of design.

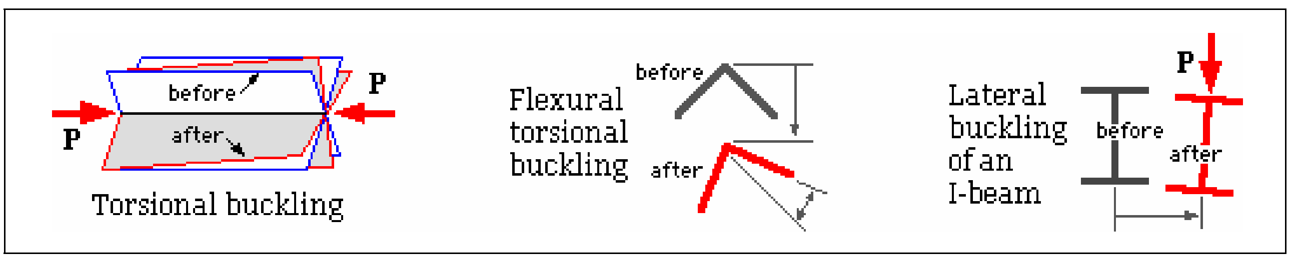


**Figure 3**

## *Buckling Load Factor*

The buckling load factor (BLF) is an indicator of the factor of safety against buckling or the ratio of the buckling loads to the currently applied loads. Table 1 illustrates the interpretation of possible BLF values returned by CosmosWorks. Since buckling often leads to bad or even catastrophic results, you should utilize a high factor of safety (FOS) for buckling loads. That is, the value of unity in Table 1 should be replaced with the FOS value.

The BLF can be quite sensitive to geometrical imperfections in the part. If buckling is a concern you should try introducing slight changes, like a dent in a thin section, to see what effect it would have. If you are using symmetry in your stress analysis model you may miss an important buckling load if you just use symmetry in the buckling study. Thus, after completing the symmetry buckling load factor calculation you should repeat the study by replacing the symmetry restraint with an anti-symmetry restraint.

**Table 1 Interpretation of the Buckling Load Factor (BLF)**

| BLF Value | Buckling Status | Remarks |
|---|---|---|
| 0 < BLF < 1 | Buckling predicted | The applied loads exceed the estimated critical loads. Buckling will occur. |
| BLF = 1 | Buckling predicted | The applied loads are exactly equal to the critical loads. Buckling is expected. |
| -1 < BLF < 0 | Buckling possible | Buckling is predicted if you reverse the load directions. |
| BLF = -1 | Buckling possible | Buckling is expected if you reverse the load directions. |
| 1 < BLF | Buckling not predicted | The applied loads are less than the estimated critical loads. |
| BLF < -1 | Buckling not predicted | The applied loads are less than the estimated critical loads, even if you reverse their directions. |

## *References*


1. W.C. Young, R.G. Budynas, *Roark's Formulas for Stress and Strain*, 7-th Ed., Mc Graw-Hill, 2002.
2. W.C. Young, R.G. Budynas, *Roark and Young on TK,* Universal Technical Systems Rockford, IL, 2002. (All 1400 design cases and 5000 equations are available with TK Solver.)
3. W.C. Young, R.G. Budynas, *Roark's Formulas on Excel,* Universal Technical Systems Rockford, IL, 2005.
4. H. Ziegler, Principles of Structural Stability, Gill-Blaisdell, Waltham, MA, 1968.


 