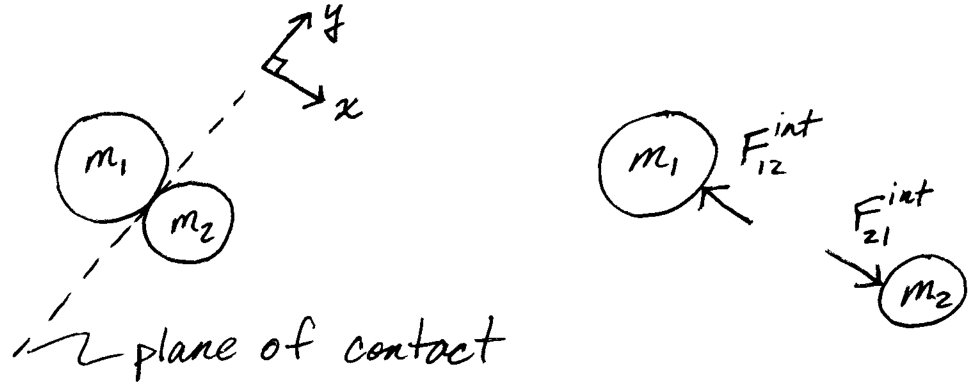
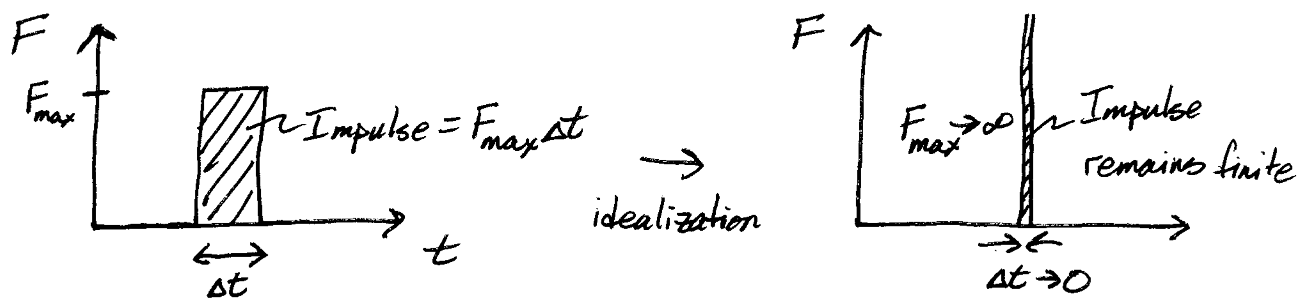
Elastic Impacts



Notice that during the impact, the internal impact forces act only perpendicular to the plane of contact.

We will approximate impact forces to act over <u>zero</u> time but impart a finite impulse to the respective mass.



Note then, that any other finite forces, e.g. gravity forces, that are acting on the mass will impart <u>zero</u> impulse to the mass <u>during</u> the instant of the impact.

Back to our two masses,

The internal forces act only in the x-direction, so for <u>each</u> mass linear momentum in the y-direction is conserved.

$$\therefore \quad v_{y1}^{i} = v_{y1}^{f} \quad \text{and} \quad v_{y2}^{i} = v_{y2}^{f}$$

Also, there are no external forces on the entire system, so linear momentum of the entire system is conserved in all directions. We are interested in the x-direction,

$$m_1 v_{x1}^{i} + m_2 v_{x2}^{i} = m_1 v_{x1}^{f} + m_2 v_{x2}^{f}$$

We still have 2 unknowns, v_{x1}^{f} and v_{x2}^{f}, for this one equation.

The final equation comes from the definition of the coefficient of restitution for the impact.

$$\boxed{e = \frac{v_{sep}}{v_{app}}}$$

Easiest to use this equation and take signs from a diagram.

v_{app} = how fast the masses approach each other $\perp$ to the plane of contact

v_{sep} = how fast the masses separate

$$v_{app} = v_{x1}^{i} - v_{x2}^{i}$$

$$v_{sep} = v_{x2}^{f} - v_{x1}^{f}$$

Note the change in sign

** The signs used here assume positive in positive x-direction

$$\therefore \boxed{e = \frac{v_{x2}^{f} - v_{x1}^{f}}{v_{x1}^{i} - v_{x2}^{i}}}$$

$e \le 1$, $e = 1 \rightarrow$ energy is conserved

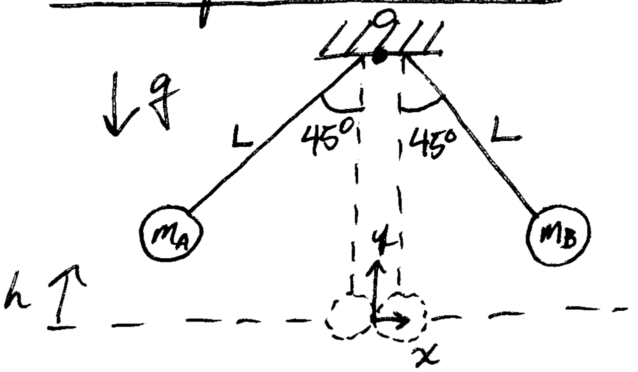
$$\therefore \; v_{x2}^{f} = v_{x1}^{f} + e\left(v_{x1}^{i} - v_{x2}^{i}\right)$$

$$m_1 v_{x1}^{i} + m_2 v_{x2}^{i} = m_1 v_{x1}^{f} + m_2 v_{x1}^{f} + m_2 e\left(v_{x1}^{i} - v_{x2}^{i}\right)$$

$$v_{x1}^{f} = \frac{m_1 v_{x1}^{i} - e m_2 v_{x1}^{i} + (1+e) m_2 v_{x2}^{i}}{m_1 + m_2}$$

$$v_{x2}^{f} = \frac{m_2 v_{x2}^{i} - e m_1 v_{x2}^{i} + (1+e) m_1 v_{x1}^{i}}{m_1 + m_2}$$

Example Problem



$m_B = 2 m_A$

$e = 0.75$

Determine the maximum angles, θ_A^{max} and θ_B^{max}, that each mass reaches after the collision.

How much energy is lost in the collision.

Prior to the collision energy is conserved for each mass, i.e.

$$KE_i + PE_i + \cancel{W^{nc}}^{\to 0} = KE_f + PE_f$$
$$0 + mgh_1 = \tfrac{1}{2}mv^2 + 0$$

$$\therefore\ v = \sqrt{2gh_1} = \sqrt{2g(L - L\cos\theta)}$$

Linear momentum in x-direction is conserved for the 2 masses combined. Remember that momentum is a vector equation so signs of velocities must be accounted for.

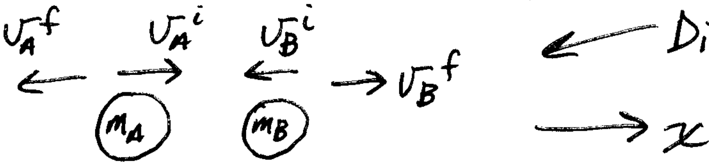


Diagram indicating positive directions.

We will take the magnitudes as positive and the directions shown above as positive.

x-momentum $\rightarrow$ $m_A v_A^i - m_B v_B^i = m_B v_B^f - m_A v_A^f$

coefficient of restitution $\rightarrow$ $e = \dfrac{v_{sep}}{v_{app}}$

$$e = 0.75 = \frac{v_B^f + v_A^f}{v_A^i + v_B^i} = \frac{v_B^f + v_A^f}{2\sqrt{2gh_1}}$$

$$v_B^f = \tfrac{3}{2}\sqrt{2gh_1} - v_A^f$$

$$\therefore \cancel{m_A}\sqrt{2gh_1} - 2\cancel{m_A}\sqrt{2gh_1} = \mathbf{2}\cancel{m_A}\left(\tfrac{3}{\mathbf{2}}\sqrt{2gh_1} - v_A^f\right) - \cancel{m_A}v_A^f$$

$$\therefore\ -4\sqrt{2gh_1} = -3v_A^f$$

$$\therefore\ v_A^f = \frac{4}{3}\sqrt{2gh_1}$$

$$v_B^f = \frac{1}{6}\sqrt{2gh_1}$$

After the collision energy is conserved for each mass, i.e.

$$\frac{1}{2}m_A(v_A^f)^2 = m_A g h_2^A$$

$$\frac{1}{2}m_A \frac{16}{9}2gh_1 = m_A g h_2^A$$

$$\rightarrow h_2^A = \frac{16}{9}h_1 \rightarrow L(1-\cos\theta_A^{max}) = L\left(1-\frac{\sqrt{2}}{2}\right)\frac{16}{9}$$

$$\therefore \boxed{\theta_A^{max} = 61.36°}$$

$$\frac{1}{2}m_B(v_B^f)^2 = m_B g h_2^B$$

$$\frac{1}{2}m_B \frac{1}{36}2gh_1 = m_B g h_2^B$$

$$h_2^B = \frac{1}{36}h_1 \rightarrow L(1-\cos\theta_B^{max}) = \frac{1}{36}L\left(1-\frac{\sqrt{2}}{2}\right)$$

$$\boxed{\theta_B^{max} = 7.31°}$$

How much energy is lost in the collision?

$E_i = E_i^A + E_i^B$ since energy is conserved for each mass prior to the collision we can evaluate E_i^A & E_i^B at any point before the collision.

$\therefore E_i = m_A g h_1 + m_B g h_1$

$E_f = E_f^A + E_f^B$ $\rightarrow$ again since energy is conserved for <u>each</u> mass after the collision we can evaluate E_f^A & E_f^B at 2 different points after the collision.

$\therefore E_f = m_A g h_2^A + m_B g h_2^B$

$E_i = 3 m_A g h_1$, $E_f = m_A g \frac{16}{9} h_1 + 2 m_A g \frac{1}{36} h_1$

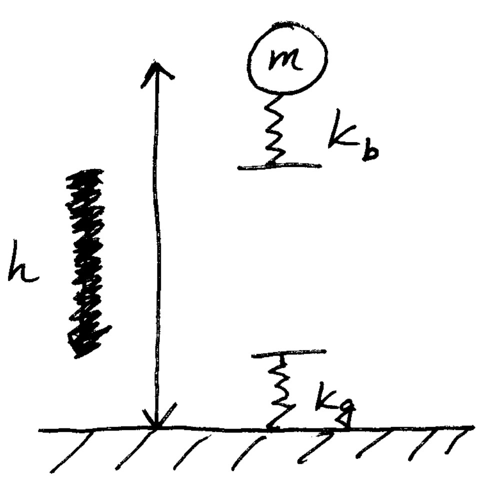
$$E_f = \frac{33}{18} m_A g h_1$$

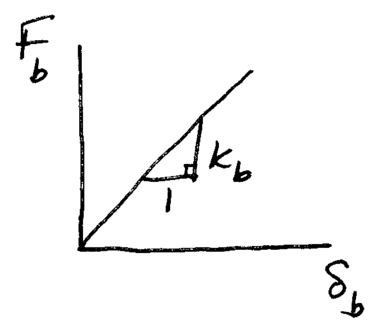
$\frac{E_f}{E_i} = \frac{33}{54}$ $\rightarrow$ 39% of energy is lost in the collision.
& 61% of energy is retained.

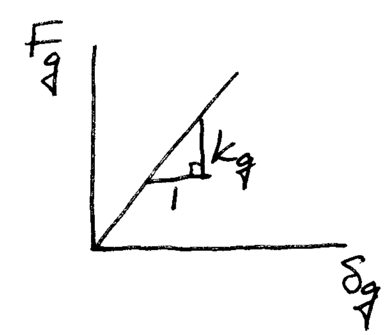
A Physical Question

What is the maximum force that a ball experiences when it hits the ground if it is dropped from a height h?

Let's model both the ball and ground with springs.







Assume energy is conserved.

$$\therefore \quad KE_i + PE_i + \cancel{W^{nc}}^{\,0} = KE_f + PE_f$$

$$0 + mgh = 0 + \tfrac{1}{2}k_b\delta_b^2 + \tfrac{1}{2}k_g\delta_g^2$$

Note that in PE_f we are going to neglect changes in height due to δ_b and δ_g, this implies that our solution is only valid if $h >> \delta_b$ and $h >> \delta_g$. A more careful analysis is required if δ_b, δ_g and h are of the same order of magnitude.

We also know that the magnitudes of the force the ground places on the ball and the force the ball places on the ground are equal, i.e.

$$F^b = F^g \rightarrow k_b \delta_b = k_g \delta_g$$

$$\therefore \quad \delta_g = \frac{k_b}{k_g} \delta_b$$

$$\rightarrow mgh = \frac{1}{2} k_b \delta_b^2 + \frac{1}{2} k_g \left(\frac{k_b}{k_g}\right)^2 \delta_b^2$$

$$= \frac{1}{2}\left(k_b + \frac{k_b^2}{k_g}\right) \delta_b^2$$

$$= \frac{1}{2}\left(\frac{k_b}{k_g}\right)(k_b + k_g)\, \delta_b^2$$

$$\delta_b = \sqrt{\frac{2mgh}{\left(\frac{k_b}{k_g}\right)(k_b + k_g)}}$$

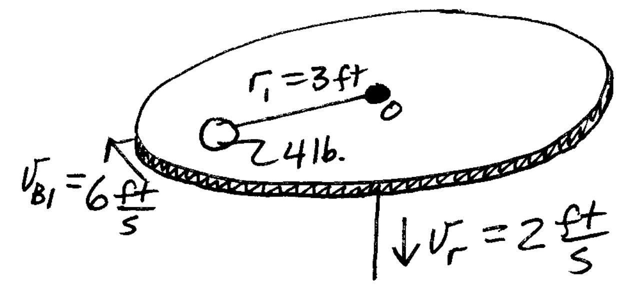
$$\therefore \quad F^b_{max} = k_b \sqrt{\frac{2mgh}{\left(\frac{k_b}{k_g}\right)(k_b + k_g)}} = \sqrt{\frac{2mgh\, k_b k_g}{k_b + k_g}}$$

Take rigid ground $\rightarrow k_g \rightarrow \infty$

$\therefore \quad F^b_{max} \rightarrow \sqrt{2mgh\, k_b}$ for rigid ground

F^b_{max} increases as height, mass of ball, or stiffness of the ball increases.

Test Problem Example



Determine ball's speed when $r_2 = 2ft$. How much work must be done to pull the cord down?
Neglect friction and the size of the ball.

-Note: $v_{r1} = -2\frac{ft}{s}$

The tension of the string acting on the ball acts through the center of the hole so there are no moments in the z-direction acting on the ball → conservation of angular momentum.

CAM about O → $-m v_{B1} r_1 = +m v_{\theta 2} r_2$

$$\therefore \quad v_{\theta 2} = -\frac{r_1}{r_2} v_{B1}$$

$$\therefore \quad v_{\theta 2} = -\frac{3}{2}\cdot 6 = -9\frac{ft}{s}$$

$$v_{r2} = -2\frac{ft}{s}$$

$$\vec{v}_2 = \left(-2\vec{e}_r - 9\vec{e}_\theta\right)\frac{ft}{s} \rightarrow v_2 = \sqrt{85}\frac{ft}{s}$$

Work done can be determined from work-energy

i.e. $KE_i + \cancel{PE_i}^{\,0} + W^{nc} = KE_f + \cancel{PE_f}^{\,0}$

$KE_i = \frac{1}{2} m v_1^2$, $\vec{v}_1 = (-2\vec{e}_r - 6\vec{e}_\theta)$ ft/s

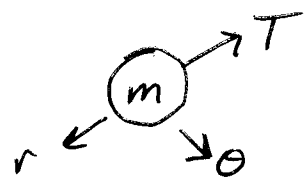
$\therefore KE_i = \frac{1}{2} m \left(40 \frac{ft^2}{s^2}\right)$

$KE_f = \frac{1}{2} m v_2^2$, $\vec{v}_2 = (-2\vec{e}_r + v_\theta \vec{e}_\theta)$ ft/s

$\therefore KE_f = \frac{1}{2} m (4 + v_\theta^2) = \frac{1}{2} m \left(85 \frac{ft^2}{s^2}\right)$

$$\therefore \boxed{W^{nc} = W^T = \frac{1}{2} m (85-40) = \frac{1}{2}\frac{4}{32.2}(45) = 2.795 \text{ ft·lb}}$$

<u>Another approach to W^T</u>



$\Sigma F_r = -T = m a_r = m(\ddot{r} + -r\dot{\theta}^2)$

But $\dot{r} = -2 \frac{ft}{s}$ = constant $\rightarrow \ddot{r} = 0$
and $r\dot{\theta}^2 = \frac{1}{r}(r\dot{\theta})^2 = \frac{v_\theta^2}{r}$

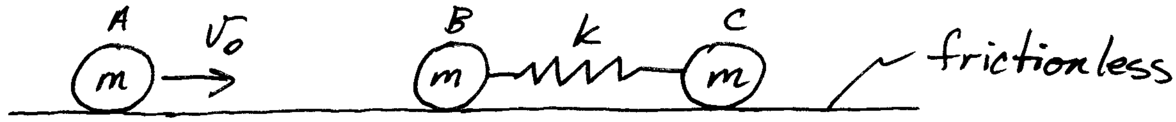
CAM $\rightarrow -m r_1 v_{B1} = m r v_\theta \rightarrow v_\theta = -\frac{r_1}{r} v_{B1}$

$\therefore \quad r\dot{\theta}^2 = \frac{r_1^2}{r^3} v_{B1}^2 = \frac{324}{r^3}$

$\therefore -T = -m \frac{324}{r^3} \rightarrow T = m \frac{324}{r^3} = \frac{4}{32.2} \frac{324}{r^3} = 40.248 \frac{1}{r^3}$

$$\therefore \boxed{W^T = \int \vec{T} \cdot d\vec{r} = -\int_{r_1}^{r_2} T\,dr = -40.248 \int_3^2 \frac{1}{r^3}\,dr = 2.795 \text{ ft·lb.}}$$

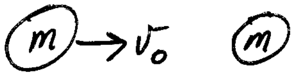
Example



$e = 0.5$, Determine the maximum deflection of the spring after the impact.

Analyze impact first

Prior



After

$$m v_0 = m v_A + m v_B$$

$$e = \frac{v_{sep}}{v_{app}} = \frac{v_B - v_A}{v_0} = 0.5$$

$$\rightarrow v_A = v_B - \frac{v_0}{2}$$

$$\cancel{m} v_0 = \cancel{m}\left(v_B - \frac{v_0}{2}\right) + \cancel{m} v_B$$

$$v_B = \frac{3}{4} v_0 \quad \rightarrow \quad v_A = \frac{1}{4} v_0$$

After impact energy of system BC is conserved.

$$KE_i + \cancel{PE_i}^{\,0} + \cancel{W^{nc}}^{\,0} = KE_f + PE_f$$

$$\frac{1}{2} m v_{Bi}^2 = \frac{1}{2} m v_{Bf}^2 + \frac{1}{2} m v_{cf}^2 + \frac{1}{2} k \delta^2$$

We know $v_{Bi} = \frac{3}{4} v_0$, but we have 3 other unknowns v_{Bf}, v_{cf} and δ.

First recognize that $\vec{v}_c = \vec{v}_B + \vec{v}_{c/B}$, or in our simple 1-D case

$$\rightarrow v_c = v_B + v_{c/B}$$

Now, consider yourself riding along on mass B. $v_{c/B}$ is then the velocity of mass C that you observe. From this perspective it is easy to see that the maximum deflection of the spring will occur when $v_{c/B} = 0$.

$\therefore$ at max δ $\quad v_{c/B} = 0 \rightarrow v_{cf} = v_{Bf}$

But we need one more equation. This equation comes from conservation of momentum for system BC after the impact.

$$\rightarrow \quad m v_{Bi} = m v_{Bf} + m v_{cf} = 2m v_{Bf}$$

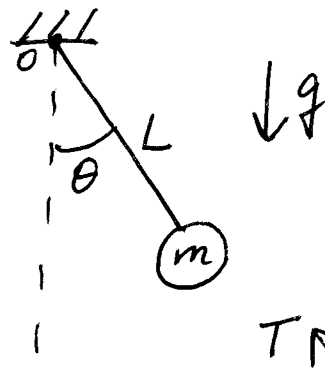
$$\rightarrow m \frac{3}{4} v_0 = 2m v_{Bf} \rightarrow v_{Bf} = \frac{3}{8} v_0$$

$$\rightarrow \frac{1}{2} m \left(\frac{3}{4} v_0\right)^2 = \frac{1}{2} m \left(\frac{3}{8} v_0\right)^2 + \frac{1}{2} m \left(\frac{3}{8} v_0\right)^2 + \frac{1}{2} k \delta_{max}^2$$

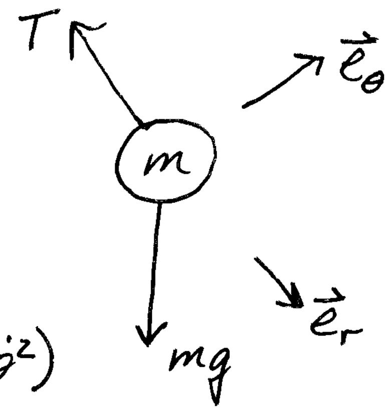
$$\delta_{max}^2 = \frac{m}{k} \frac{9}{32} v_0^2 \rightarrow \boxed{\delta_{max} = v_0 \frac{3}{4\sqrt{2}} \sqrt{\frac{m}{k}}}$$

A

Three Methods for the Equation of Motion for a Pendulum



<u>F=ma method</u> FBD



$$\Sigma F_r = -T + mg\cos\theta = m(\ddot{r} - r\dot{\theta}^2)$$

$$\Sigma F_\theta = -mg\sin\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

Single dof is θ, $r = L$, $\dot{r} = 0$, $\ddot{r} = 0$

$$F_r \rightarrow \quad mg\cos\theta - T = -mL\dot{\theta}^2$$

$$F_\theta \rightarrow \quad -mg\sin\theta = mL\ddot{\theta}$$

$$\therefore \quad \boxed{\ddot{\theta} + \frac{g}{L}\sin\theta = 0}$$ Equation of motion for θ

(B)

<u>Energy method</u> $KE_i + PE_i = KE_f + PE_f$

or if we are looking at two instants in time infinitesimally close to one another we have

$$\frac{d}{dt}(KE + PE) = 0$$

$KE = \frac{1}{2}mv^2 = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2)$, recall $r = L$, $\dot{r} = 0$

$PE = -mgL\cos\theta$ (take reference for height at point O)

$$\rightarrow \frac{d}{dt}\left(\frac{1}{2}mL^2\dot{\theta}^2 - mgL\cos\theta\right) = 0$$

$$\frac{1}{2}mL^2 \underbrace{2\dot{\theta}\ddot{\theta}} + mgL\sin\theta\,\dot{\theta} = 0$$

$$\frac{d}{dt}(\dot{\theta}^2) = \frac{d}{d\dot{\theta}}\frac{d\dot{\theta}}{dt}(\dot{\theta}^2) = 2\dot{\theta}\frac{d\dot{\theta}}{dt} = 2\dot{\theta}\ddot{\theta}$$

then cancelling $mL\dot{\theta} \rightarrow L\ddot{\theta} + g\sin\theta = 0$

$$\boxed{\ddot{\theta} + \frac{g}{L}\sin\theta = 0}$$

c

Momentum method

Note that the tension T does not place a moment about point O, but the force due to gravity, mg, does and therefore angular momentum is not conserved about O.

However, we can use $\vec{M}_O = \dfrac{d\vec{h}_o}{dt}$ (O is fixed)

$$\vec{M}_o = -mg\underbrace{L\sin\theta}_{\text{moment arm}}\,\vec{e}_z$$

$$\vec{h}_o = \vec{r}\times m\vec{v} = L\vec{e}_r \times m\left(\overset{0}{\cancel{\dot{r}}}\vec{e}_r + \overset{L}{r}\dot{\theta}\vec{e}_\theta\right)$$

$$= mL^2\dot{\theta}\,\vec{e}_z$$

$$\rightarrow\ -mgL\sin\theta\,\vec{e}_z = \frac{d}{dt}\left(mL^2\dot{\theta}\,\vec{e}_z\right)$$

$$= mL^2\ddot{\theta}\,\vec{e}_z + mL^2\dot{\theta}\overset{0}{\cancel{\frac{d\vec{e}_z}{dt}}}$$

$$\rightarrow\ \boxed{\ddot{\theta} + \frac{g}{L}\sin\theta = 0}$$

Particle Dynamics Review

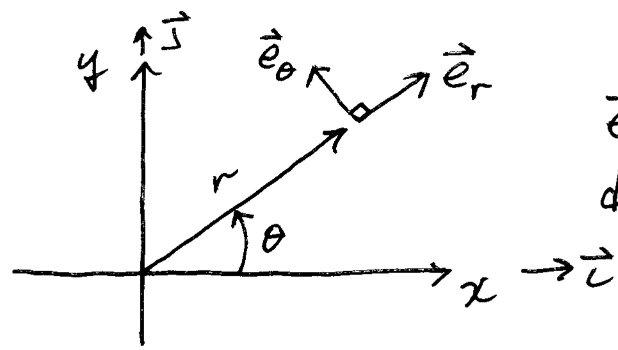
Kinematics

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} \qquad \text{Cartesian}$$
$$= R\vec{e}_R \quad \text{or} \quad r\vec{e}_r \qquad \text{Polar}$$
$$= R\vec{e}_R + z\vec{e}_z \qquad \text{Cylindrical}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k} \qquad \text{Cartesian}$$
$$= \dot{R}\vec{e}_R + R\dot{\theta}\vec{e}_\theta \qquad \text{Polar}$$
$$= \dot{R}\vec{e}_R + R\dot{\theta}\vec{e}_\theta + \dot{z}\vec{e}_z \qquad \text{Cylindrical}$$
$$= \dot{s}\vec{e}_t \qquad \text{Path}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k} \qquad \text{Cartesian}$$
$$= (\ddot{R} - R\dot{\theta}^2)\vec{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\vec{e}_\theta \qquad \text{Polar}$$
$$= (\ddot{R} - R\dot{\theta}^2)\vec{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\vec{e}_\theta + \ddot{z}\vec{e}_z \qquad \text{Cylindrical}$$
$$= \ddot{s}\vec{e}_t + \frac{\dot{s}^2}{\rho}\vec{e}_n \qquad \text{Path}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(\frac{dx}{dy}\right)^2\right]^{3/2}}{\left|\frac{d^2x}{dy^2}\right|}$$



$\vec{e}_\theta$ always points in the direction which θ increases

$$\vec{e}_r = \vec{i}\cos\theta + \vec{j}\sin\theta$$
$$\vec{e}_\theta = -\vec{i}\sin\theta + \vec{j}\cos\theta$$

$$\vec{i} = \vec{e}_r\cos\theta - \vec{e}_\theta\sin\theta$$
$$\vec{j} = \vec{e}_r\sin\theta + \vec{e}_\theta\cos\theta$$

The following are useful if $a_x = a_x(x)$

$$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\frac{dx}{dt} = v_x \frac{dv_x}{dx}$$

$$\rightarrow \quad \int_A^B a_x(x)\,dx = \int_A^B v_x\,dv_x = \tfrac{1}{2}(v_x^B)^2 - \tfrac{1}{2}(v_x^A)^2$$

Similarly : $$\int_A^B \ddot{\theta}(\theta)\,d\theta = \int_A^B \dot{\theta}\,d\dot{\theta} = \tfrac{1}{2}(\dot{\theta}_B)^2 - \tfrac{1}{2}(\dot{\theta}_A)^2$$

<u>Constant</u> acceleration motion in a fixed (non-rotating) direction:

$$a = \text{const}$$

$$v = at + v_0$$

Assuming x is the direction of interest $\longrightarrow$ $$x = \tfrac{1}{2}at^2 + v_0 t + x_0$$

Constraint Equations: $$\frac{d}{dt}f(\theta) = \frac{df}{d\theta}\frac{d\theta}{dt} = \frac{df}{d\theta}\dot{\theta}$$

$$\frac{d}{dt}\left[\frac{d}{dt}f(\theta)\right] = \frac{d}{dt}\left(\frac{df}{d\theta}\right)\dot{\theta} + \frac{df}{d\theta}\frac{d}{dt}(\dot{\theta})$$

$$= \frac{d}{d\theta}\left(\frac{df}{d\theta}\right)\underbrace{\frac{d\theta}{dt}}_{\dot{\theta}}\dot{\theta} + \frac{df}{d\theta}\ddot{\theta}$$

$$\frac{d}{dt}\left[\frac{d}{dt}f(\theta)\right] = \frac{d^2 f}{d\theta^2}\dot{\theta}^2 + \frac{df}{d\theta}\ddot{\theta}$$

(C)

<u>Kinetics</u> $\vec{F} = m\vec{a} \rightarrow \Sigma F_x = ma_x = m\ddot{x}$

$$\Sigma F_y = ma_y = m\ddot{y}$$

$$\Sigma F_z = ma_z = m\ddot{z}$$

$$\Sigma F_R = ma_R = m(\ddot{R} - R\dot{\theta}^2)$$

$$\Sigma F_\theta = ma_\theta = m(R\ddot{\theta} + 2\dot{R}\dot{\theta})$$

<u>Work-Energy</u> $W_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$

$d\vec{r} = \vec{i}\,dx + \vec{j}\,dy + \vec{k}\,dz$ Cartesian

$= \vec{e}_R\,dR + \vec{e}_\theta\,R\,d\theta + \vec{e}_z\,dz$ Polar

$= \vec{e}_t\,ds$ Path

$d\vec{r} = \vec{v}\,dt$ in general

$$KE_i + PE_i + W_{NC} = KE_f + PE_f$$

$KE = \frac{1}{2}mv^2 = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2)$ Cartesian

$= \frac{1}{2}m(v_R^2 + v_\theta^2 + v_z^2)$ Cylindrical

$PE = mgh$ gravity

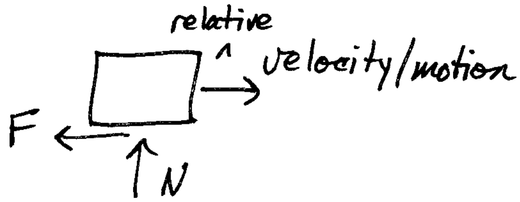
$-\frac{Gm_1m_2}{r}$ Gravitation

$\frac{1}{2}k(L-L_0)^2 = \frac{1}{2}k\delta^2$ linear spring $\rightarrow F_s = k\delta$

$\frac{1}{2}k(\theta-\theta_0)^2$ angular spring $\rightarrow C_s = k\theta$

D

W_{NC} accounts for any other forces in the system not included in PE. For example, the work due to friction is always negative and only exists if there is relative motion between surfaces.



$\rightarrow W_{AB}^{friction} = -\int_A^B \mu_k N \, ds$

Note: Forces normal to the motion do <u>zero</u> work.

<u>Impulse - Momentum</u>

$$\int_{t_1}^{t_2} \sum_{i=1}^{n} \vec{F}_i^{ext} \, dt = \sum_{i=1}^{n} \left(m_i \vec{v}_{i,2} - m_i \vec{v}_{i,1} \right)$$

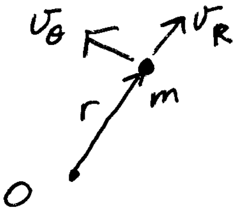
This can be applied to a system of particles, $n>1$, or a single particle $n=1$.

Most useful for analyzing impacts.

<u>Angular Impulse - Angular momentum</u>

$\vec{h}^o = \vec{r}_{m/o} \times m\vec{v}$ for a single particle

Most useful:



$$h_z^o = m r v_\theta = mr(r\dot{\theta}) = mr^2\dot{\theta}$$

$$\int_{t_1}^{t_2} \sum_{i=1}^{n} \vec{M}_i^{ext,A} \, dt = \sum_{i=1}^{n} \left(\vec{h}_{i,2}^{A} - \vec{h}_{i,1}^{A} \right)$$

* where A is a fixed point or A=CM

E

Impacts

Assumed to occur over zero time but have a finite impulse. Other finite forces will have zero impulse during the impact.

Coefficient of restitution : $e = \dfrac{v_{sep}}{v_{app}}$

$\xrightarrow{v_{Ai}}$ (A) (B) $\xleftarrow{v_{Bi}}$ $\qquad\qquad$ $\xleftarrow{v_{Af}}$ (A) (B) $\xrightarrow{}$ v_{Bf}

before : $v_{App} = v_{Ai} + v_{Bi}$ $\qquad\qquad$ after : $v_{sep} = v_{Af} + v_{Bf}$

Conservation of momentum $\rightarrow m_A v_{Ai} - m_B v_{Bi} = -m_A v_{Af} + m_B v_{Bf}$

Another sign convention

$\xrightarrow{v_{Ai}}$ (A) (B) $\xrightarrow{v_{Bi}}$ $\qquad\qquad$ $\xrightarrow{v_{Af}}$ (A) (B) $\rightarrow v_{Bf}$

before : $v_{App} = v_{Ai} - v_{Bi}$ $\qquad\qquad$ after : $v_{sep} = v_{Bf} - v_{Af}$

Cons. mom. $\rightarrow$ $m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$