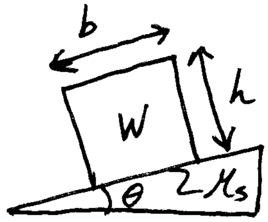
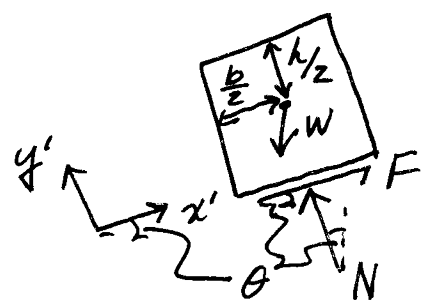
Tipping versus Sliding

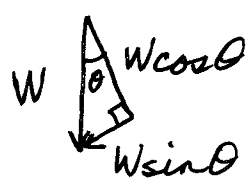


Consider a block of weight W with base b and height h. The coefficient of friction between the block and the inclined plane is μ_s.

If we increase the angle of the inclined plane gradually from $\theta = 0°$, what are the conditions required for the block to slide before tipping and visa versa?

FBD of the block.





Analysis: $\Sigma F_{x'} = F - W\sin\theta = 0$

$\therefore F = W\sin\theta$

$\Sigma F_{y'} = N - W\cos\theta = 0$

$\therefore N = W\cos\theta$

For impending sliding we have $F = \mu_s N$

$\therefore \quad W \sin\theta = \mu_s W \cos\theta$

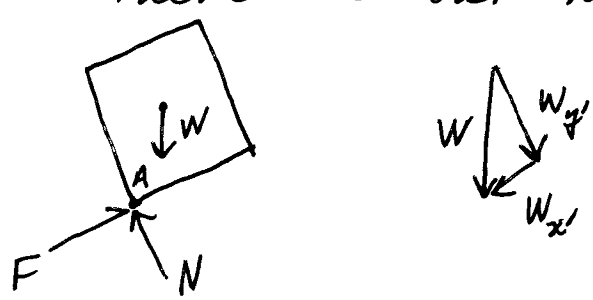
$\mu_s = \tan\theta$ for impending sliding

if $\mu_s < \tan\theta \rightarrow$ slides

$\mu_s > \tan\theta \rightarrow$ equilibrium

What about tipping?

Note that when the block is just about to tip both the normal force and the friction force act at the corner of the block because the remainder of the block is just about to lift off of the inclined plane. Therefore our new FBD looks like



Let's analyze the moments about point A.

$$\Sigma M_{z'}^{A} = +W_{x'}\frac{h}{2} - W_{y'}\frac{b}{2} = 0$$

$$\frac{h}{2}\cancel{W}\sin\theta - \frac{b}{2}\cancel{W}\cos\theta = 0$$

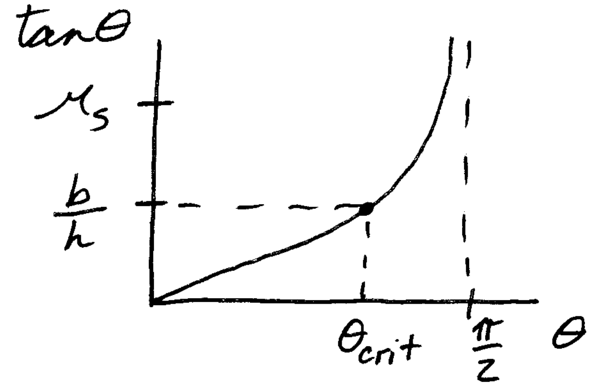
$$\tan\theta = \frac{b}{h} \text{ for impending tipping.}$$

$$\frac{b}{h} > \tan\theta \rightarrow \text{equilibrium}$$

$$\frac{b}{h} < \tan\theta \rightarrow \text{tipping}$$

Now let's combine our two conditions.

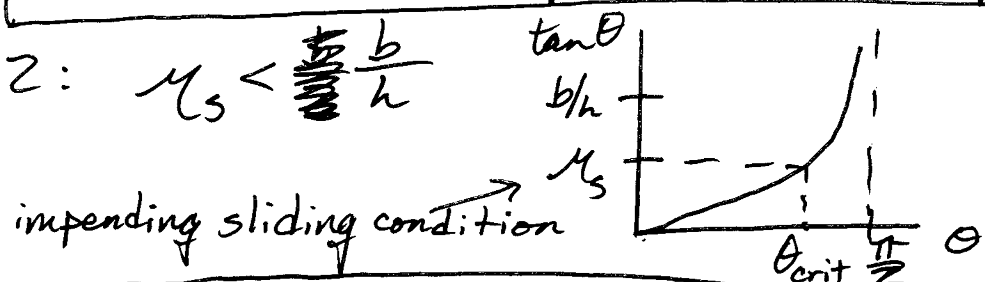
Case 1: $\mu_s > \frac{b}{h}$



The non-equilibrium impending tipping condition is met first, therefore

if $\boxed{\mu_s > \frac{b}{h} \rightarrow \text{tipping before sliding}}$

Case 2: $\mu_s < \frac{b}{h}$

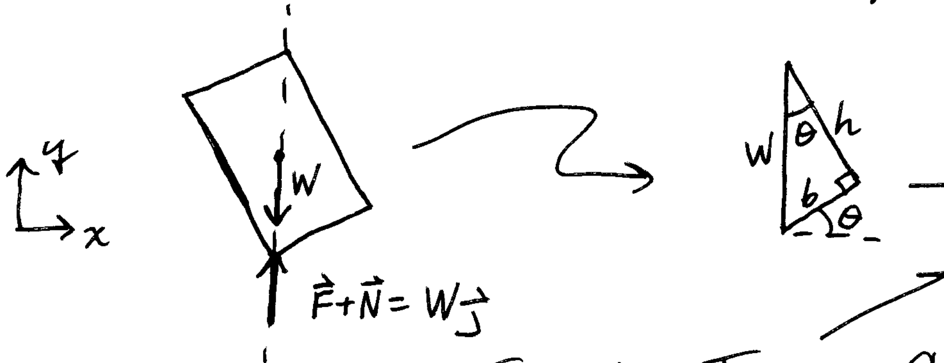


if $\boxed{\mu_s < \frac{b}{h} \rightarrow \text{sliding before tipping}}$

These conditions make sense because for "sticky" surfaces, i.e. μ_s large, we expect tipping, and for short stubby blocks, i.e. $\frac{b}{h}$ large, we expect sliding.

- A geometric analysis of tipping.

Recall that for impending tipping we know exactly where both the normal and friction forces act, i.e. right at the corner. Since both of these forces act at the same point we can consider them as one force. Then our block is just a 2-force body, the weight W and the resultant contact force at the corner ($\vec{F}+\vec{N}$). We know that for a two force body the forces must be equal and opposite and act through the same point of application. Since the contact force acts through the corner, then so does the weight at impending tipping.



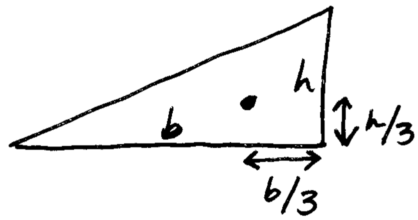
$$\boxed{\tan\theta = \frac{b}{h}}$$

Impending Tipping Condition

What if the block was triangular instead of rectangular?

The important geometric parameters are the location of c.m.

For a right triangle the center of mass is located at



Back to our problem.

Looks like the rectangle but with c.m. at $\frac{h}{3}$ and $b/3$ instead of $\frac{h}{2}$ and $\frac{b}{2}$

$\therefore$ tipping at $\tan\theta = \frac{b/3}{h/3} = \frac{b}{h}$

Looks like the rectangle but with c.m. at $\frac{h}{3}$ and $\frac{2b}{3}$ instead of $\frac{h}{2}$ and $\frac{b}{2}$.

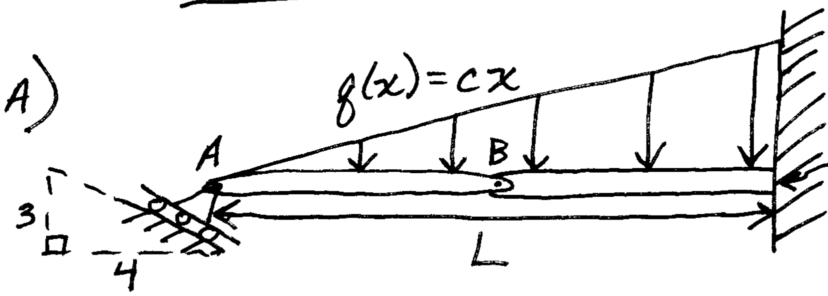
$\therefore$ tipping at $\tan\theta = \frac{2b/3}{h/3} = \frac{2b}{h}$

Therefore, as we expect, the block on the left will tend to tip before the block on the right.

Note the the sliding analysis did not depend on b or h (or W for that matter) so the sliding condition for the triangles will still be when $\tan\theta = \mu_s$.

Problems from 2001 Test

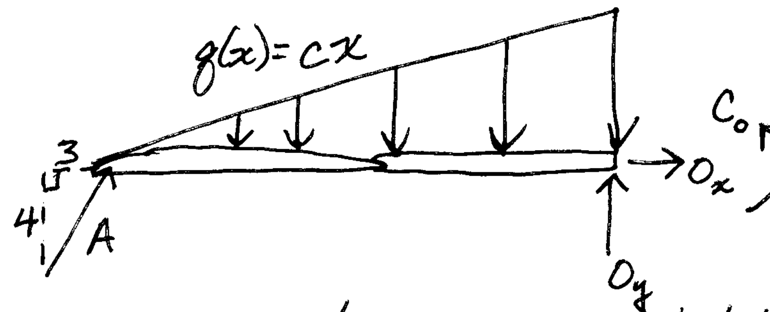
A)



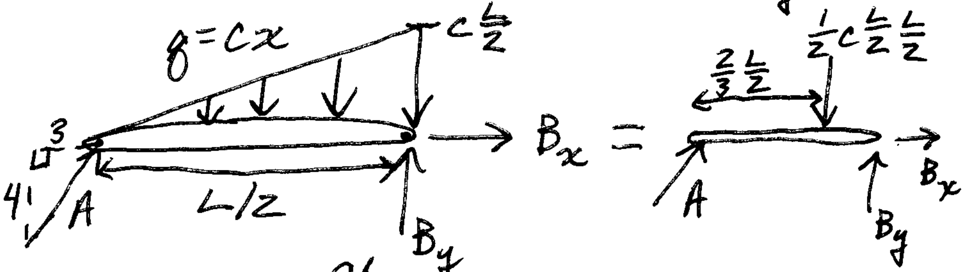
Reactions at O and A?

FBD's

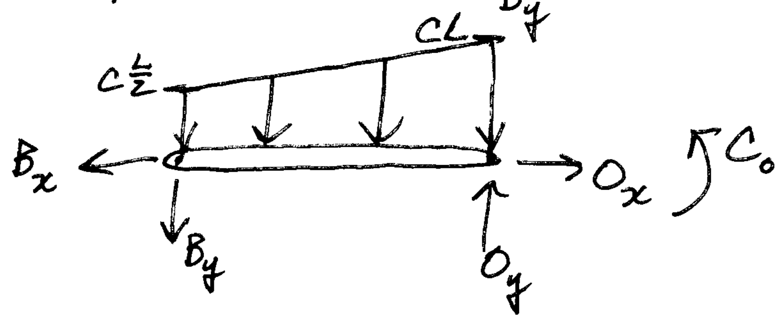
Entire structure:



AB :



BO :



Analysis : Entire structure → 4 unknowns A, O_x, O_y, C_o
3 equations

AB : 3 unkowns A, B_x, B_y
3 equations

BO : 5 unknowns B_x, B_y, O_x, O_y, C_o
3 equations

Analyze AB first: $\Sigma F_x = \frac{3}{5}A + B_x = 0$

$$\Sigma F_y = \frac{4}{5}A - \frac{1}{8}cL^2 + B_y = 0$$

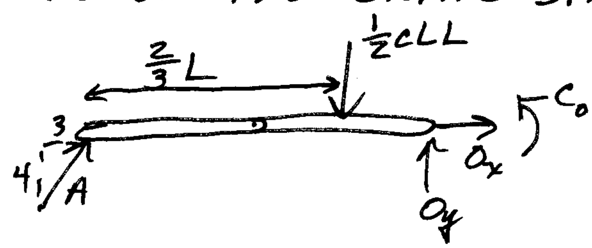
$$\Sigma M_z^B = \underbrace{-\frac{4}{5}A}_{A_y}\frac{L}{2} + \frac{1}{8}cL^2\frac{L}{6} = 0$$

$$\therefore \boxed{A = \frac{5}{96}cL^2}$$

$$\rightarrow B_x = -\frac{3}{96}cL^2, \quad B_y = \frac{8}{96}cL^2$$

Now analyze either BO or the entire structure.

Entire structure:



$$\Sigma F_x = \frac{3}{5}A + O_x = 0 \rightarrow O_x = -\frac{3}{5}A = \frac{-3}{96}cL^2$$

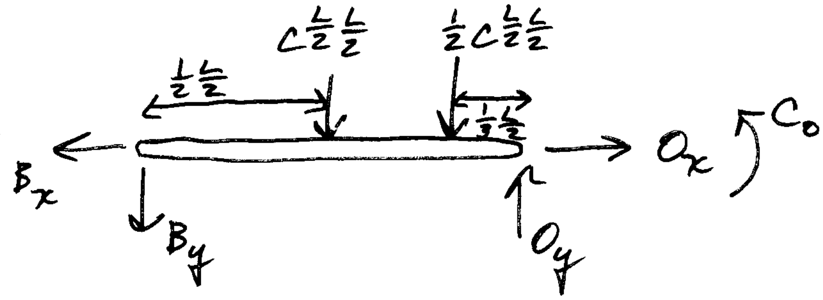
$$\Sigma F_y = \frac{4}{5}A - \frac{1}{2}cL^2 + O_y \rightarrow O_y = \frac{1}{2}cL^2 - \frac{4}{96}cL^2 = \frac{44}{96}cL^2$$

$$\Sigma M_z^o = C_o + \frac{1}{2}cL^2\frac{L}{3} - \frac{4}{5}AL = 0$$

$$C_o = \frac{4}{5}\frac{5}{96}cL^2L - \frac{1}{6}cL^3 = \frac{-12}{96}cL^3$$

Now we can check this solution by analyzing BO.

BO :



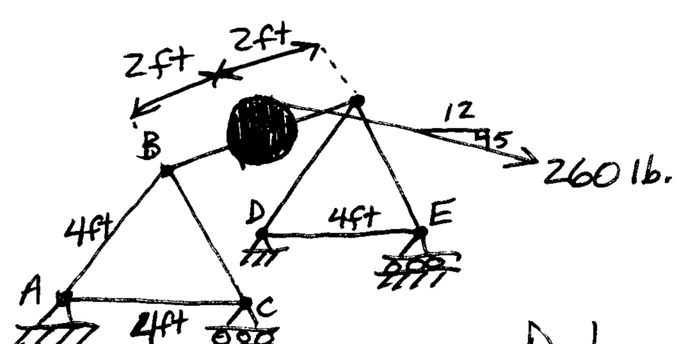
$$\Sigma F_x = -B_x + O_x = +\frac{3}{96}cL^2 - \frac{3}{96}cL^2 = 0 \checkmark$$

$$\Sigma F_y = -B_y - \frac{1}{4}cL^2 - \frac{1}{8}cL^2 + O_y = \frac{-8}{96}cL^2 - \frac{24}{96}cL^2 - \frac{12}{96}cL^2 + \frac{44}{96}cL^2 = 0 \checkmark$$

$$\Sigma M_z^o = C_o + B_y\frac{L}{2} + \frac{1}{4}cL^2\frac{L}{4} + \frac{1}{8}cL^2\frac{L}{6} = \frac{-12}{96}cL^3 + \frac{4}{96}cL^3 + \frac{6}{96}cL^3 + \frac{2}{96}cL^3 = 0 \checkmark$$

$$\therefore \boxed{\begin{array}{l} A = \frac{5}{96}cL^2 \text{ or } A_x = \frac{3}{96}cL^2,\ A_y = \frac{4}{96}cL^2 \\ O_x = \frac{-3}{96}cL^2,\ O_y = \frac{44}{96}cL^2 \\ C_o = \frac{-12}{96}cL^3 \end{array}}$$

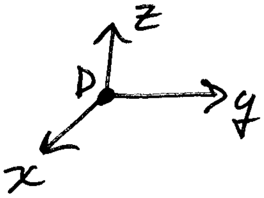
B)



Radius of spool = 1ft.
Weight of spool = 900 lb.

Determine the moment about the axis through C and D due to both the weight of the spool and the 260 lb. tension.

Choose a coordinate system $\rightarrow$

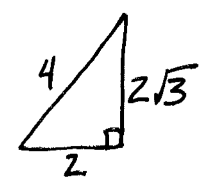


$$\vec{M}_{CD} = (\vec{M}_D \cdot \vec{e}_{CD})\,\vec{e}_{CD}$$

$$\vec{M}_D = \vec{r}_{T/D} \times \vec{T} + \vec{r}_{W/D} \times \vec{W}$$

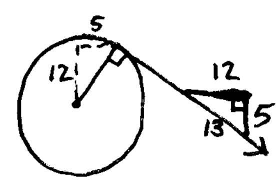
$$\vec{e}_{CD} = \frac{4\vec{i} + 4\vec{j}}{\sqrt{16+16}} = \frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j}$$

$$\vec{W} = -900\,\vec{k}$$



$$\vec{r}_{W/D} = 2\vec{i} + 2\vec{j} + 2\sqrt{3}\,\vec{k}$$

$$\vec{r}_{T/D} = 2\vec{i} + \left(2 + \tfrac{5}{13}\right)\vec{j} + \left(2\sqrt{3} + \tfrac{12}{13}\right)\vec{k}$$



$$\vec{T} = 260\left(\tfrac{12}{13}\vec{j} - \tfrac{5}{13}\vec{k}\right)$$
$$= 240\,\vec{j} - 100\,\vec{k}$$

$$\therefore\ \vec{M}_D = \left.\begin{aligned} &480\overbrace{\vec{i}\times\vec{j}}^{\vec{k}} - 200\overbrace{\vec{i}\times\vec{k}}^{-\vec{j}} - 100\left(2+\tfrac{5}{13}\right)\overbrace{\vec{j}\times\vec{k}}^{\vec{i}} \\ &+ 240\left(2\sqrt{3}+\tfrac{12}{13}\right)\underbrace{\vec{k}\times\vec{j}}_{-\vec{i}} \end{aligned}\right\}\ \vec{r}_{T/D}\times\vec{T}$$

$$\left.+\ -1800\underbrace{\vec{i}\times\vec{k}}_{-\vec{j}} - 1800\underbrace{\vec{j}\times\vec{k}}_{\vec{i}}\ \right\}\ \vec{r}_{W/D}\times\vec{W}$$

Then $\vec{M}_D \cdot \vec{e}_{CD} = \frac{\sqrt{2}}{2}\left[-100\left(2+\tfrac{5}{13}\right) - 240\left(2\sqrt{3}+\tfrac{12}{13}\right) - 1800\right]$

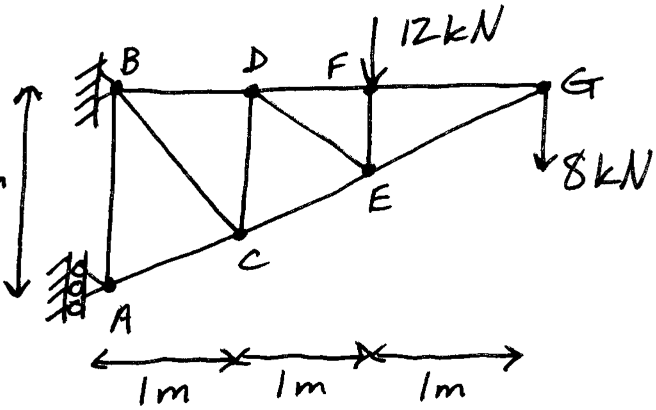
$$+ \frac{\sqrt{2}}{2}\left[200 + 1800\right] = -771.7 \text{ lb}\cdot\text{in}$$

$$\rightarrow \vec{M}_{CD} = -771.7\left(\tfrac{\sqrt{2}}{2}\vec{i} + \tfrac{\sqrt{2}}{2}\vec{j}\right) \text{ lb}\cdot\text{in}$$

$$\boxed{\vec{M}_{CD} = -545.7\,\vec{i} - 545.7\,\vec{j} \text{ lb}\cdot\text{in.}}$$

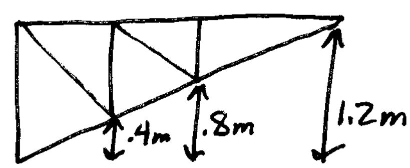
* A better choice of origin would have been directly under the center of the spool. This point is still on the axis CD, but the moment due to the weight W about this point is zero.

c)

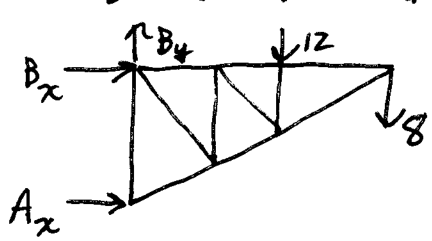


Forces carried by each member? Method of sections for DF, EC, DE.

Some geometry

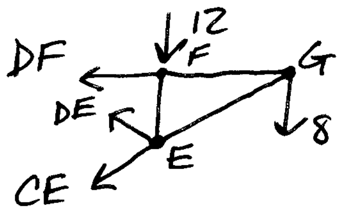


FBD of structure:



$$\Sigma F_x = 0 = A_x + B_x \quad \rightarrow \quad B_x = -40 \text{ kN}$$
$$\Sigma F_y = B_y - 12 - 8 = 0 \rightarrow B_y = 20 \text{ kN}$$
$$\Sigma M_z^B = 1.2 A_x - 2\cdot 12 - 3\cdot 8 = 0 \rightarrow A_x = 40 \text{ kN}$$

Section through DF, EC, DE



$$\Sigma M_z^E = -8 + 0.4\,DF = 0$$
$$\rightarrow DF = 20\,kN \text{ (tension)}$$
$$\Sigma F_x = -DF - DE\frac{1}{\sqrt{1.16}} - CE\frac{1}{\sqrt{1.16}} = 0$$

$$\Sigma F_y = -12 - 8 + DE\frac{.4}{\sqrt{1.16}} - CE\frac{.4}{\sqrt{1.16}} = 0$$

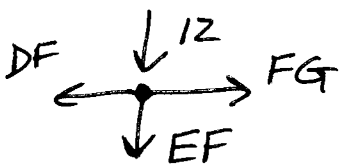
$$\therefore CE = DE - 20\frac{\sqrt{1.16}}{.4}$$

$$\rightarrow -20 - DE\frac{1}{\sqrt{1.16}} - DE\frac{1}{\sqrt{1.16}} + 20\frac{\sqrt{1.16}}{.4}\frac{1}{\sqrt{1.16}} = 0$$
$$\therefore DE = 20\left(-1 + \frac{1}{.4}\right)\frac{\sqrt{1.16}}{2} = 16.16\,kN \text{ (tension)}$$

$$\rightarrow CE = -37.7\,kN \text{ (compression)}$$

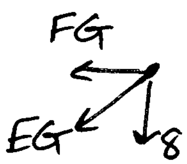
Method of joints on the rest

F :



$$\Sigma F_y = -12 - EF = 0 \rightarrow EF = -12\,(c)$$
$$\Sigma F_x = FG - DF = 0$$
$$\rightarrow FG = DF = 20\,(t)$$

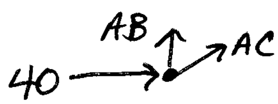
G :



$$\Sigma F_x = -FG - EG\frac{1}{\sqrt{1.16}} = 0$$

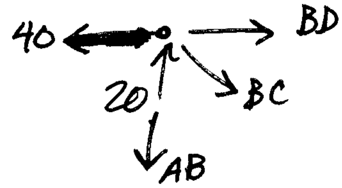
$$\rightarrow EG = -20\sqrt{1.16} = -21.54\,(c)$$

A :



$$\Sigma F_x = 40 + AC\frac{1}{\sqrt{1.16}} = 0 \rightarrow AC = -40\sqrt{1.16} = -43.08\,(c)$$
$$\Sigma F_y = AB + AC\frac{.4}{1.16} = 0 \rightarrow AB = +40 \cdot .4 = +16\,(t)$$

B:



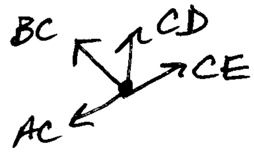
$$\Sigma F_x = BD - 40 + BC\frac{1}{\sqrt{1.64}} = 0$$

$$\Sigma F_y = -AB + 20 - BC\frac{.8}{\sqrt{1.64}} = 0$$

$$\rightarrow BC = 4\,\frac{\sqrt{1.64}}{.8} = 6.4\ (t)$$

$$\rightarrow BD = 40 - \frac{4}{0.8} = 35\ (t)$$

C:



$$\Sigma F_x = CE\frac{1}{\sqrt{1.16}} - AC\frac{1}{\sqrt{1.16}} - BC\frac{1}{\sqrt{1.64}} = 0$$

$$= -37.7\frac{1}{\sqrt{1.16}} + 40\sqrt{1.16}\frac{1}{\sqrt{1.16}} - \frac{4}{.8}\frac{\sqrt{1.64}}{\sqrt{1.64}} = 0$$

$$= -35 + 40 - 5 = 0 \checkmark$$

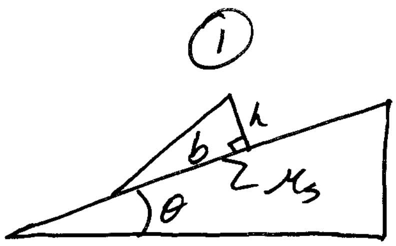
$$\Sigma F_y = CD + CE\frac{.4}{\sqrt{1.16}} - AC\frac{.4}{\sqrt{1.16}} + BC\frac{.8}{\sqrt{1.64}} = 0$$

$$\rightarrow CD = 35(.4) - 40(.4) - 4 = -6\ (c)$$

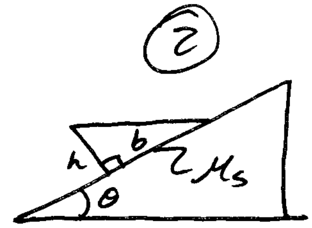
All values are in kN.

AB = 16 kN (tension)
AC = −43.1 kN (compression)
BC = 6.4 kN (tension)
BD = 35 kN (tension)
CD = −6 kN (compression)
CE = −37.7 kN (compression)
DE = 16.16 kN (tension)
DF = 20 kN (tension)
EF = −12 kN (compression)
EG = −21.5 kN (compression)
FG = 20 kN (tension)

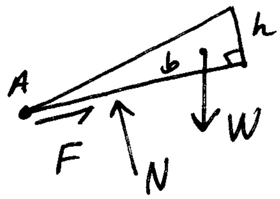
D)

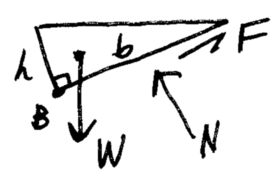


versus



FBDs





In both cases W acts at the centroid of the triangle which is located at $\frac{b}{3}$ and $\frac{h}{3}$ from the corner with the right angle.

Analysis:

$$\left.\begin{aligned} \Sigma F_{x'} &= F - W\sin\theta = 0 \\ \Sigma F_{y'} &= N - W\cos\theta = 0 \end{aligned}\right\} \text{True for case 1 or 2}$$

$$\therefore \quad F = W\sin\theta \;, \quad N = W\cos\theta$$

Impending Sliding occurs if $F = \mu_s N$

$\therefore$ impending sliding when $\tan\theta = \mu_s$

$\tan\theta < \mu_s \rightarrow$ no sliding

At impending tipping both F and N act through the corner farthest down the inclined plane.

$\therefore$ Case 1 $\quad \sum M_{z'}^{A} = -\frac{2}{3} b W\cos\theta + \frac{h}{3} W\sin\theta = 0$

$\rightarrow \tan\theta = \frac{2b}{h}$ for impending tipping

Case 2 $\quad \sum M_{z'}^{B} = -\frac{1}{3} b W\cos\theta + \frac{h}{3} W\sin\theta = 0$

$\rightarrow \tan\theta = \frac{b}{h}$ for impending tipping

Case 1 : $\mu_s > \frac{2b}{h} \rightarrow$ tips first

$\mu_s < \frac{2b}{h} \rightarrow$ slides first

Case 2 : $\mu_s > \frac{b}{h} \rightarrow$ tips first

$\mu_s < \frac{b}{h} \rightarrow$ slides first