MECH 211: Engineering Mechanics

A) Handout and go over syllabus

B) Background on Newton & mechanics

C) Background on Mechanical Engineering

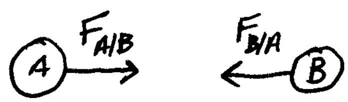
D) Newton's Laws of Motion

I) A body at rest will remain at rest unless acted upon by a force. Similarly, a body in motion will remain in motion with a constant velocity unless acted upon by a force.

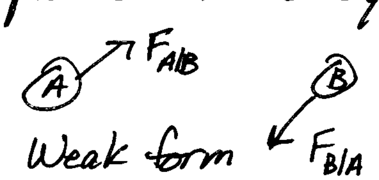
II) A body acted upon by a force will accelerate in the direction of the applied force. The magnitude of the acceleration is proportional to the force and inversely proportional to the mass of the body.

i.e. $\vec{F} = m\vec{a}$

III) For every action there is an equal and opposite reaction. The force that body A imparts on body B is equal and opposite to the force body B imparts on body A.



Strong form (colinear)



Weak form

E) Newton also devised a Law of Universal Gravitation in order to explain celestial motions.

$$F = G \frac{m_A m_B}{R^2}$$

The direction is colinear with the line connecting the center of mass of bodies A and B and is always an attraction.

F) <u>Units</u>

SI : mass (kg), length (m), time (s)
[also current (A) and liminosity]

US : force (lb), length (ft), time (s)

SI force : $\vec{F} = m\vec{a}$

$$F \Rightarrow M \frac{L}{T^2} \Rightarrow kg \frac{m}{s^2} = N$$

US mass : $m = F/a$

$$M \Rightarrow F/L \, T^2 \Rightarrow \frac{lb \, s^2}{ft} = 1 \text{ slug}$$

* The most important point about units/dimensions is that they must be consistent.

e.g. $\vec{F} = m\vec{a} + L$ where L is a length is nonsense.

Problem 1.5) $KE = \frac{1}{2}mv^2 + \frac{1}{2}mk^2\omega^2$

ω = radians/s , Determine dimensions for KE and k.

Each term in the equation must have the same dimensions.

$\therefore KE \Rightarrow \frac{1}{2}mv^2 \Rightarrow M\left(\frac{L}{T}\right)^2 \Rightarrow \frac{ML}{T^2}L \Rightarrow FL$

$\frac{ML^2}{T^2} \Rightarrow \frac{1}{2}mk^2\omega^2 \Rightarrow M\left(\frac{1}{T}\right)^2 k^2$

$\therefore k^2 \Rightarrow L^2$ or $\boxed{\begin{array}{l} k \Rightarrow L \\ KE \Rightarrow FL \text{ or } \frac{ML^2}{T^2} \end{array}}$

<u>Dimensional Analysis for Calculus expressions</u>

treat differentiation like division and integration like multiplication

e.g. $v = \frac{dx}{dt} = \frac{d}{dt}(x) \Rightarrow \frac{x}{t} \Rightarrow \frac{L}{T}$

$a = \frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d}{dt}\left[\frac{d}{dt}(x)\right] \Rightarrow \frac{x}{t \cdot t} \Rightarrow \frac{L}{T^2}$

* ↳ Do not make the mistake of $\frac{d^2x}{dt^2} \Rightarrow \frac{x^2}{t^2}$ ✗

$x = \int_{t_0}^{t_f} v\,dt \Rightarrow v \cdot t \Rightarrow \frac{L}{T} \cdot T \Rightarrow L$ ✓

Vector Algebra

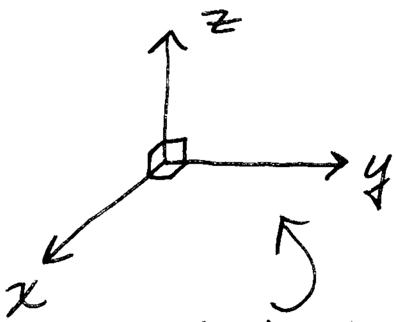
Scalars: A scalar quantity has magnitude only. e.g. temperature, voltage, time, mass, charge

Vectors: A vector quantity has both magnitude and direction. e.g. velocity, acceleration, force, electric field

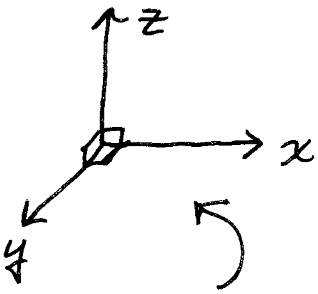
In order to specify a vector both its magnitude <u>and</u> direction must be given.

Graphically we can draw a directed line segment with the length proportional to the magnitude of the quantity being depicted.

In a more quantitative way we can give the <u>components</u> of a vector in 3 mutually exclusive directions. Furthermore, the simplest way to choose these 3 directions is to make them perpendicular to one another or orthogonal. Finally, we will also always choose these 3 directions to be right-handed.



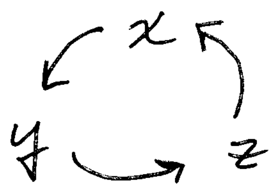
Right-handed



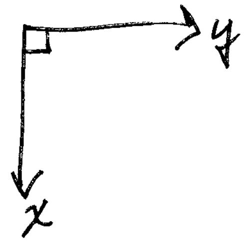
Left-handed

We will always choose this type of system.

Using your right hand, make a hitch-hiking thumb with your fingers curling from x to y then your thumb is in the z-direction. Curl your fingers from y to z and your thumb will be in the x-direction. Finally, curl from z to x and your thumb should be in the y-direction.



So if I choose x and y as follows



, then the z-direction will be out of the page for a right-handed system.

Back to the specification of a vector:

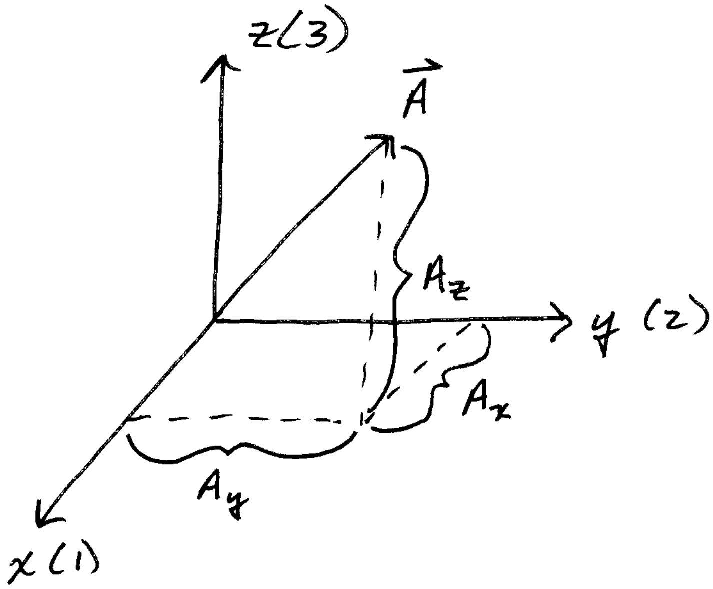
Let $\vec{e}_x$, $\vec{e}_y$ and $\vec{e}_z$ be unit vectors in the x, y and z directions. By unit vector we mean that its magnitude is 1. Then any vector $\vec{A}$ can be written as:

$$\vec{A} = A_x \vec{e}_x + A_y \vec{e}_y + A_z \vec{e}_z$$

or if we take $x \to 1$, $y \to 2$, $z \to 3$

$$\vec{A} = A_1 \vec{e}_1 + A_2 \vec{e}_2 + A_3 \vec{e}_3 = \sum_{i=1}^{3} A_i \vec{e}_i$$

A_x, A_y and A_z, (i.e. A_1, A_2, A_3), are the components of the vector $\vec{A}$ in the x, y, z (1,2,3) directions.



$\vec{A}$ can also be written as $\vec{A} = A\vec{e}_A$ where A is the magnitude of $\vec{A}$ and $\vec{e}_A$ is a unit vector in the same direction as $\vec{A}$.

If we know the components of $\vec{A}$ how can we determine its magnitude A?

Before we answer this let's look at scalar-vector multiplication and dot products.

<u>Scalar-vector multiplication</u>

$$\vec{B} = c\vec{A} \quad , \quad \vec{A} = A_x\vec{e}_x + A_y\vec{e}_y + A_z\vec{e}_z = A\vec{e}_A$$

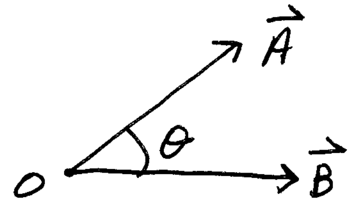
then $\vec{B} = cA_x\vec{e}_x + cA_y\vec{e}_y + cA_z\vec{e}_z = cA\,\vec{e}_A$

the middle equality indicates
$$B_x = cA_x$$
$$B_y = cA_y$$
$$B_z = cA_z$$

because $\vec{B} = B_x\vec{e}_x + B_y\vec{e}_y + B_z\vec{e}_z$

the last equality indicates the magnitude of $\vec{B}$, $B = cA$, and its direction is the same as the direction of $\vec{A}$, i.e. $\vec{e}_A$.

Dot Product (Scalar Product)



Two vectors $\vec{A}$ and $\vec{B}$ can always be used to define a plane. Just like 3 points can be used to define a plane. Let one of these points lie at the tails of these vectors, i.e. point O, another at the tip of $\vec{A}$ and the last at the tip of $\vec{B}$. Then the angle <u>in this plane</u> between $\vec{A}$ and $\vec{B}$ is θ. The dot product is defined as

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

Note that the dot product is commutative, i.e.

$$\vec{B} \cdot \vec{A} = BA\cos\theta = AB\cos\theta = \vec{A} \cdot \vec{B}$$

Let's analyze this in component form:

$$\vec{A} \cdot \vec{B} = (A_x \vec{e}_x + A_y \vec{e}_y + A_z \vec{e}_z) \cdot (B_x \vec{e}_x + B_y \vec{e}_y + B_z \vec{e}_z)$$

$$\begin{aligned} = & A_x B_x \vec{e}_x \cdot \vec{e}_x + A_x B_y \vec{e}_x \cdot \vec{e}_y + A_x B_z \vec{e}_x \cdot \vec{e}_z \\ & + A_y B_x \vec{e}_y \cdot \vec{e}_x + A_y B_y \vec{e}_y \cdot \vec{e}_y + A_y B_z \vec{e}_y \cdot \vec{e}_z \\ & + A_z B_x \vec{e}_z \cdot \vec{e}_x + A_z B_y \vec{e}_z \cdot \vec{e}_y + A_z B_z \vec{e}_z \cdot \vec{e}_z \end{aligned}$$

but $\vec{e}_x \cdot \vec{e}_x = 1$ b/c $\theta = 0$ and $|e_x| = 1$
similarly $\vec{e}_y \cdot \vec{e}_y = \vec{e}_z \cdot \vec{e}_z = 1$

also $\vec{e}_x \cdot \vec{e}_y = \vec{e}_y \cdot \vec{e}_x = 0$ b/c $\theta = 90°$

similarly $\vec{e}_x \cdot \vec{e}_z = \vec{e}_z \cdot \vec{e}_x = \vec{e}_y \cdot \vec{e}_z = \vec{e}_z \cdot \vec{e}_y = 0$

$$\therefore \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = \sum_{i=1}^{3} A_i B_i$$

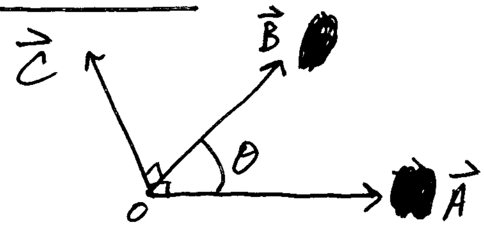
Now we can use the scalar-vector multiplication and the dot product to determine the magnitude of a vector.

Recall : $\vec{A} = A_x \vec{e}_x + A_y \vec{e}_y + A_z \vec{e}_z = A \vec{e}_A$

$$\vec{A} \cdot \vec{A} = A_x^2 + A_y^2 + A_z^2 = A^2 \underbrace{\vec{e}_A \cdot \vec{e}_A}_{1}$$

$$\therefore A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$
$$= \sqrt{\sum_{i=1}^{3} A_i^2}$$

Cross Product (Vector Product)



Again, define $\vec{A}$, $\vec{B}$ and θ as was done for the dot product. Then the vector $\vec{C} = \vec{A} \times \vec{B}$ is perpendicular to both $\vec{A}$ and $\vec{B}$ (and therefore any vector in the same plane as $\vec{A}$ and $\vec{B}$), has magnitude

$$C = AB \sin\theta$$

and positive direction using the right-hand rule sweeping $\vec{A}$ into $\vec{B}$.

Determining the components of $\vec{C}$.

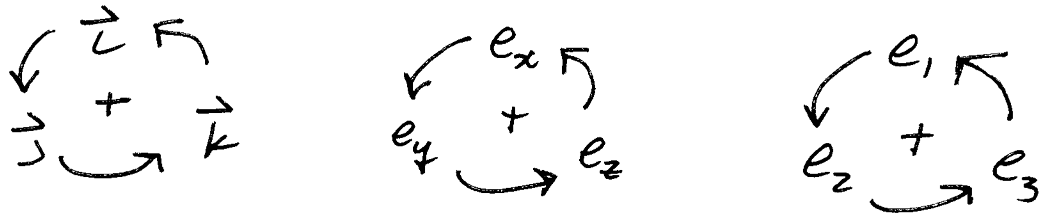
$$\vec{C} = \vec{A} \times \vec{B} = (A_x \vec{i} + A_y \vec{j} + A_z \vec{k}) \times (B_x \vec{i} + B_y \vec{j} + B_z \vec{k})$$

$$\begin{aligned} &= A_x B_x \vec{i}\times\vec{i} + A_x B_y \vec{i}\times\vec{j} + A_x B_z \vec{i}\times\vec{k} \\ &+ A_y B_x \vec{j}\times\vec{i} + A_y B_y \vec{j}\times\vec{j} + A_y B_z \vec{j}\times\vec{k} \\ &+ A_z B_x \vec{k}\times\vec{i} + A_z B_y \vec{k}\times\vec{j} + A_z B_z \vec{k}\times\vec{k} \end{aligned}$$

but note that $\vec{i}\times\vec{i} = \vec{j}\times\vec{j} = \vec{k}\times\vec{k} = 0$
b/c $\theta = 0$ and $\sin\theta = 0$

$\vec{i}\times\vec{j} = \vec{k}$ $\quad$ $\vec{j}\times\vec{k} = \vec{i}$ $\quad$ $\vec{k}\times\vec{i} = \vec{j}$

$\vec{j}\times\vec{i} = -\vec{k}$ $\quad$ $\vec{k}\times\vec{j} = -\vec{i}$ $\quad$ $\vec{i}\times\vec{k} = -\vec{j}$

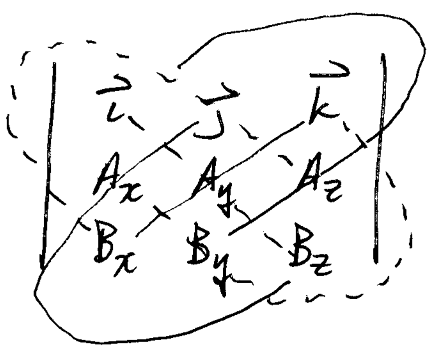


then $\vec{C} = (A_y B_z - A_z B_y)\vec{i} + (A_z B_x - A_x B_z)\vec{j}$
$+ (A_x B_y - A_y B_x)\vec{k}$

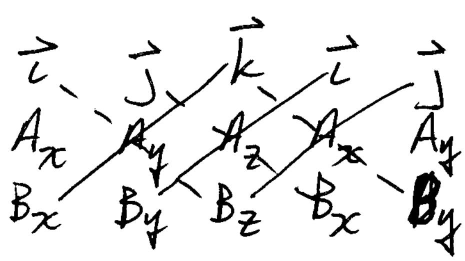
also note that if $\vec{D} = \vec{B} \times \vec{A}$ then $\vec{D} = -\vec{C}$, i.e. the cross product is <u>not</u> commutative.

<u>Matrix Method</u>

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y)\vec{i} + (A_z B_x - A_x B_z)\vec{j} + (A_x B_y - A_y B_x)\vec{k}$$



or



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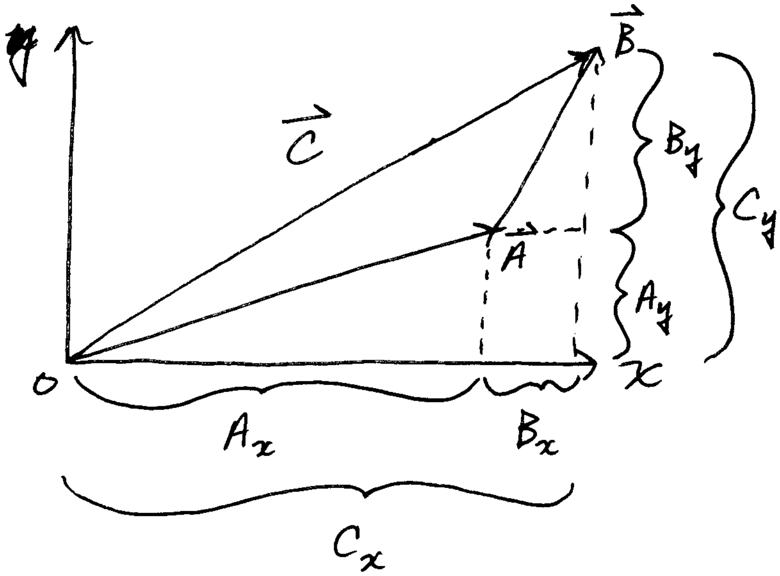
Addition



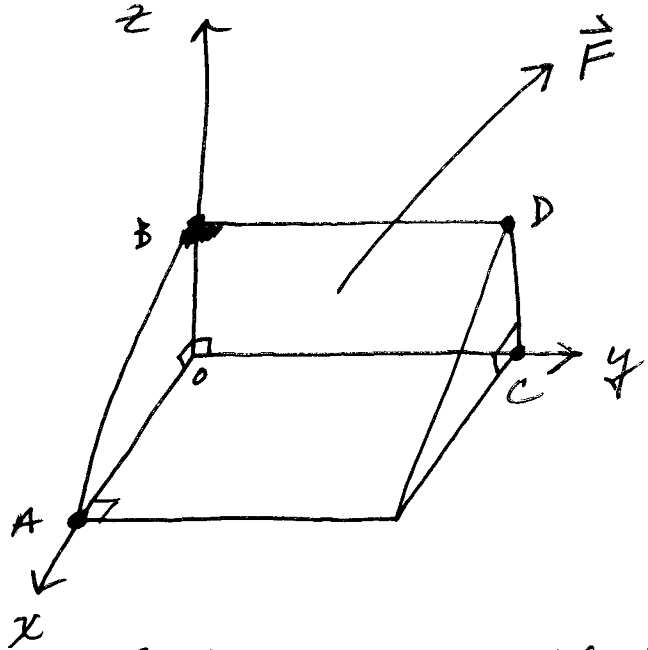
$$\vec{C} = \vec{A} + \vec{B} = \underbrace{(A_x + B_x)}_{C_x}\vec{e}_x + \underbrace{(A_y + B_y)}_{C_y}\vec{e}_y + \underbrace{(A_z + B_z)}_{C_z}\vec{e}_z$$

or $\quad C_i = A_i + B_i \quad , \; i = 1,2,3$ or x,y,z

We have already used this in our discussion of dot and cross product.

* Note that 1 vector equation constitutes $\underline{\underline{3}}$ scalar equations.

Supplement to Vector Analysis Lecture



Given: components of $\vec{F}$ and distances OA, OB, OC

Determine the part of the force $\vec{F}$ $\perp$ to the inclined plane and the part // to the plane.
parallel

First recognize that $\vec{F}$ can be decomposed into a vector component $\perp$ to the plane plus a vector component // to the plane.

i.e. $\vec{F} = \vec{F_\perp} + \vec{F_{\parallel}}$

Now, recognize that there are infinitely many directions // to the plane, but only 1 (2 if you count its opposite) $\perp$ to it. Therefore, it will be easier to determine $\vec{F_\perp}$ first.

We can always write $\vec{F_\perp}$ as $\vec{F}_\perp = F_\perp \vec{e}_\perp$. In most cases we assume $F_\perp$ is the magnitude of $\vec{F}_\perp$ and is therefore positive. However, in this type of problem it is possible (50/50 chance) to pick $\vec{e}_\perp$ in the wrong direction, so we will allow $F_\perp$ to carry a sign as well.

$F_\perp$ is the amount of $\vec{F}$ in the $\vec{e}_\perp$ direction.

$$\rightarrow \quad F_\perp = \underbrace{\vec{F} \cdot \vec{e}_\perp}$$

note this quantity has a sign & is not necessarily >0.

$$\text{then} \quad \vec{F}_\perp = \overbrace{(\vec{F} \cdot \underbrace{\vec{e}_\perp) \, \vec{e}_\perp}}^{\text{sign \& magnitude}}$$

direction (w/sign)

note that signs will "cancel" out, so it does not matter what direction we choose for $\vec{e}_\perp$

How do we get $\vec{e}_\perp$? Ans: Pick any two vectors in the plane and cross them, then $\underbrace{\text{normalize}}$ the result.

i.e. make it a unit vector

For example, two vectors in the plane are from A to B and from A to D. Note A to C is not in the plane.

A to B : $\vec{AB} = -OA\,\vec{i} + OB\,\vec{k}$

A to D : $\vec{AD} = -OA\,\vec{i} + OC\,\vec{j} + OB\,\vec{k}$

$$\vec{AB} \times \vec{AD} = OA^2 \underbrace{\vec{i} \times \vec{i}}_{0} - OA \cdot OC \underbrace{\vec{i} \times \vec{j}}_{\vec{k}} - OA \cdot OB \underbrace{\vec{i} \times \vec{k}}_{-\vec{j}}$$
$$- OB \cdot OA \underbrace{\vec{k} \times \vec{i}}_{\vec{j}} + OB \cdot OC \underbrace{\vec{k} \times \vec{j}}_{-\vec{i}} + OB^2 \underbrace{\vec{k} \times \vec{k}}_{0}$$

$$= - OB \cdot OC \, \vec{i} + \underbrace{(OA \cdot OB - OB \cdot OA)}_{0} \vec{j} - OA \cdot OC \, \vec{k}$$

note: this vector goes <u>into</u> the plane, but ultimately it does not matter. Also at this point it would be OK to realize this and change the sign, but not necessary.

Now: $$\vec{e}_\perp = \frac{- OB \cdot OC \, \vec{i} - OA \cdot OC \, \vec{k}}{\sqrt{(OB \cdot OC)^2 + (OA \cdot OC)^2}}$$

Now it is possible to compute $\vec{F}_\perp$ as

$$\vec{F}_\perp = \left(-F_x \cdot OB \cdot OC - F_z \cdot OA \cdot OC \right) \frac{-OB \cdot OC \, \vec{i} - OA \cdot OC \, \vec{k}}{(OB \cdot OC)^2 + (OA \cdot OC)^2}$$

Finally, $$\vec{F}_{\parallel} = \underset{\text{given}}{\vec{F}} - \underset{\text{just computed}}{\vec{F}_\perp}$$

How might you verify that $\vec{F}_{\parallel}$ is in the plane?

Ans: $\vec{F}_{\parallel} \cdot \vec{e}_\perp$ should be equal to zero.

Concurrent Forces

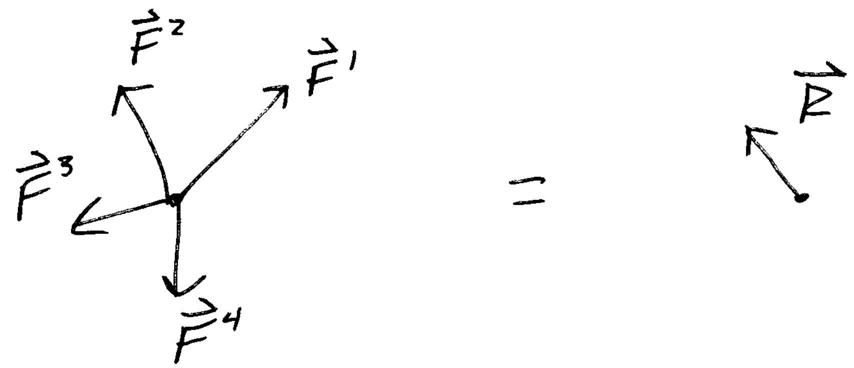
~~[illegible]~~ Force is a vector quantity.

i.e. $\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$

The resultant of multiple forces acting at the same point is simply the sum of those forces.

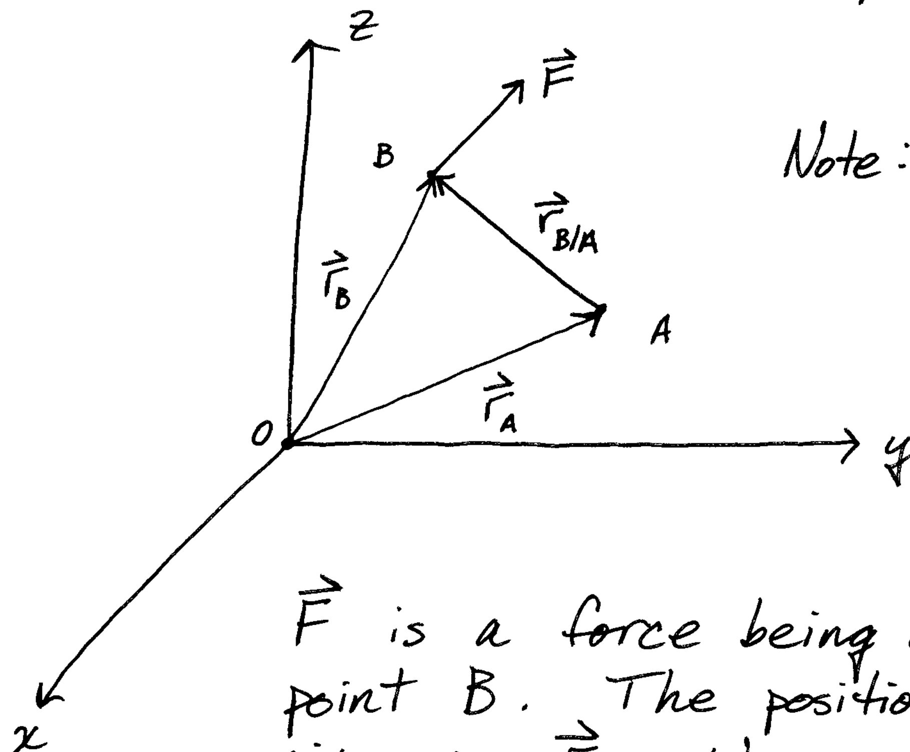
i.e. $\vec{R} = \sum_{i=1}^{N} \vec{F}^i$

were there are a total of N forces acting at the point and $\vec{F}^i$ is the i^{th} force.



Moments (a.k.a. Torques)

- Moment of a force about a point



Note: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

$\vec{F}$ is a force being applied at point B. The position of B is given as $\vec{r}_B$. We want the moment due to the force $\vec{F}$ about the point A located at position $\vec{r}_A$.

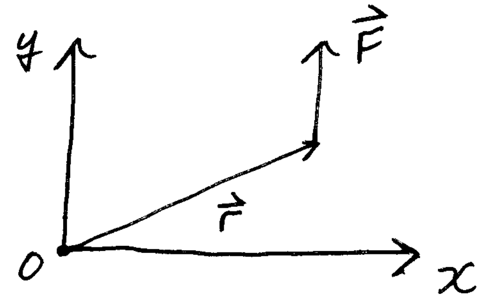
The position of B with respect to A is denoted $\vec{r}_{B/A}$ and is a vector from A to B.

then $\boxed{\vec{M}_A = \vec{r}_{B/A} \times \vec{F}}$

One point of view for understanding moments is as follows. The moment due to a force tells us how strongly the force $\vec{F}$ tends to rotate the moment arm $\vec{r}_{B/A}$ about point A.

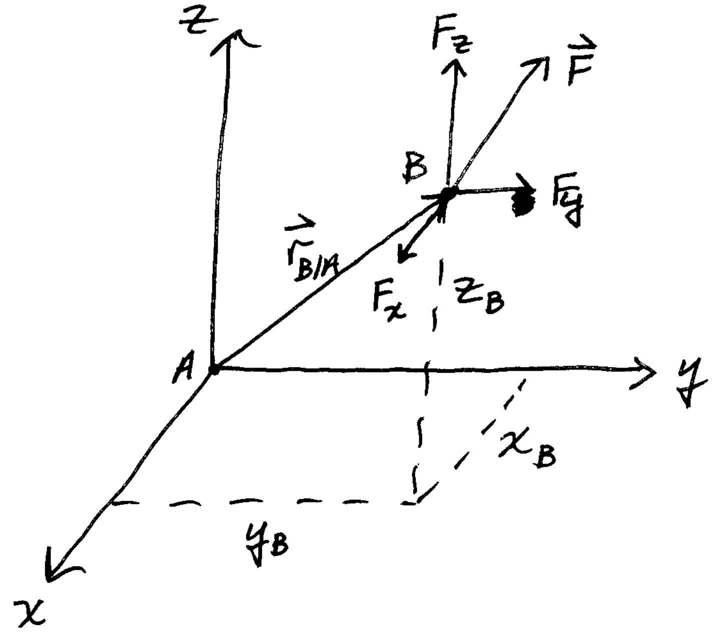
Note that if $\vec{F}$ is in the same direction as $\vec{r}_{B/A}$ then the moment arm is tending to be stretched but not rotated. Hence, only the part of the force that is perpendicular to the moment arm can give rise to a moment.

A point about direction:



Here $\vec{F}$ tends to rotate the moment arm $\vec{r}$ counterclockwise. Using the right hand rule this corresponds to the positive z-direction.

Let's move our coordinate system to point A.



$$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$$

$$\vec{r}_{B/A} = x_B \vec{i} + y_B \vec{j} + z_B \vec{k}$$

note F_x is perpendicular to the moment arms with length z_B and y_B.

F_x tends to rotate the z_B moment arm about the y-axis in a positive sense and the y_B moment arm about the z-axis in a negative sense. Hence the F_x component of the force contributes to the moment as follows

$$F_x : \quad F_x z_B \vec{j} - F_x y_B \vec{k}$$

similarly

$$F_y : \quad F_y x_B \vec{k} - F_y z_B \vec{i}$$

$$F_z : \quad F_z y_B \vec{i} - F_z x_B \vec{j}$$

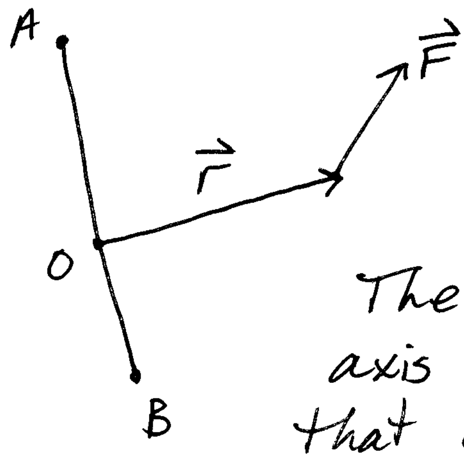
in total $\rightarrow$
$$\vec{M}_A = (F_z y_B - F_y z_B)\vec{i} + (F_x z_B - F_z x_B)\vec{j} + (F_y x_B - F_x y_B)\vec{k}$$

By the matrix method

$$\vec{M}_A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_B & y_B & z_B \\ F_x & F_y & F_z \end{vmatrix} = F_z y_B \vec{i} + F_y x_B \vec{k} + F_x z_B \vec{j} - F_x y_B \vec{k} - F_z x_B \vec{j} - F_y z_B \vec{i}$$

$$\vec{M}_A = (F_z y_B - F_y z_B)\vec{i} + (F_x z_B - F_z x_B)\vec{j} + (F_y x_B - F_x y_B)\vec{k}$$

Moment of a force about an axis.



O is <u>any</u> point on the axis.

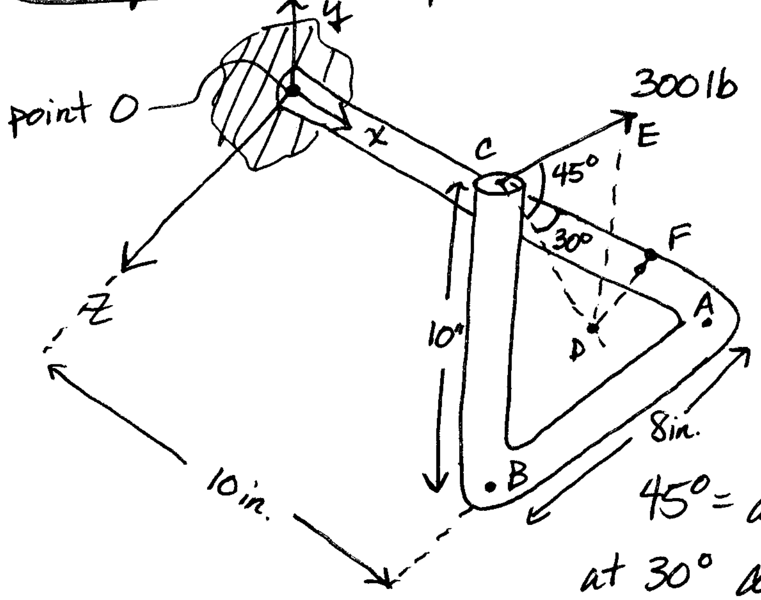
The moment of $\vec{F}$ about the axis AB is a <u>vector</u> quantity that gives the amount of the moment due to $\vec{F}$ about any point on the axis in the direction about the axis.

$\vec{M}_o$ = moment of $\vec{F}$ about O

then $$\vec{M}_{AB} = \underbrace{\left(\vec{M}_o \cdot \vec{e}_{AB}\right)}_{\pm\text{magnitude}} \underbrace{\vec{e}_{AB}}_{\text{direction}}$$

or $$\vec{M}_{AB} = \left(\underbrace{\vec{r} \times \vec{F}}_{\vec{M}_o} \cdot \vec{e}_{AB}\right) \vec{e}_{AB}$$

Example : Test problem from Fall 2000



30° = angle between the projection of the force onto the x-z plane and the x-axis.

45° = angle between dashed line at 30° and force.

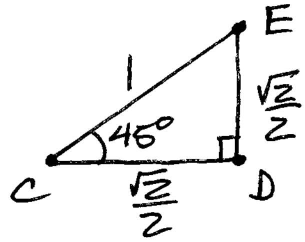
Determine moment about point O and moment about the axis from O to B.

Moment about $O = \vec{M}_o = \vec{r}_{c/o} \times \vec{F}$

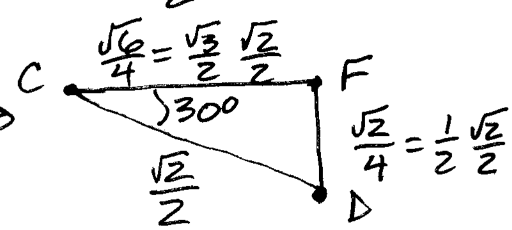
$$\vec{r}_{c/o} = 10\,in.\,\vec{i} + 10\,in.\,\vec{j} + 8\,in.\,\vec{k}$$

$$\vec{F} = F\,\vec{e}_F = 300\,lbs.\,\vec{e}_F$$

Consider triangle CDE $\rightarrow$



Now consider triangle CDF $\rightarrow$



$$\therefore\ \vec{e}_F = \frac{\sqrt{6}}{4}\vec{i} + \frac{\sqrt{2}}{2}\vec{j} + \frac{\sqrt{2}}{4}\vec{k}$$

Check to be sure $\vec{e}_F$ is a unit vector.

$$\vec{e}_F \cdot \vec{e}_F = \left(\frac{\sqrt{6}}{4}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{4}\right)^2 = \frac{6}{16} + \frac{8}{16} + \frac{2}{16} = \frac{16}{16} \checkmark$$

$$\vec{M}_o = (10\vec{i} + 10\vec{j} + 8\vec{k})\text{ in.} \times \left(300\frac{\sqrt{6}}{4}\vec{i} + 300\frac{\sqrt{2}}{2}\vec{j} + 300\frac{\sqrt{2}}{4}\vec{k}\right)\text{lb.}$$

$$= \left(3000\frac{\sqrt{2}}{4} - 2400\frac{\sqrt{2}}{2}\right)\vec{i} + \left(2400\frac{\sqrt{6}}{4} - 3000\frac{\sqrt{2}}{4}\right)\vec{j}$$

$$+ \left(3000\frac{\sqrt{2}}{2} - 3000\frac{\sqrt{6}}{4}\right)\vec{k}$$

$$\vec{M}_o = (-636\vec{i} + 409\vec{j} + 284\vec{k})\text{ lb}\cdot\text{in}$$

Moment about axis OB $= \vec{M}_{OB} = (\vec{M}_o \cdot \vec{e}_{OB})\vec{e}_{OB}$

$$\vec{e}_{OB} = \frac{10\vec{i} + 8\vec{k}}{\sqrt{164}} = 0.78\vec{i} + 0.62\vec{k}$$

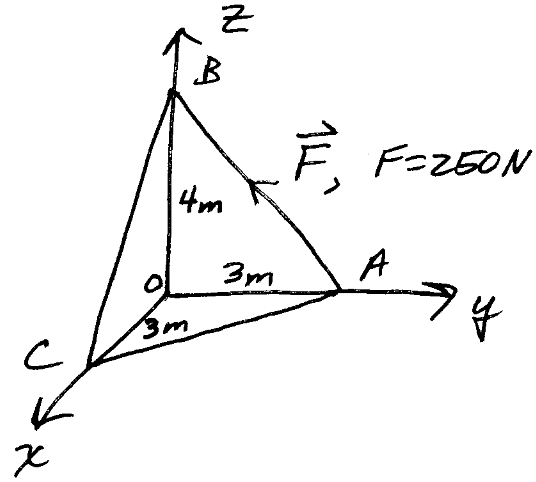
$(164)^{.5}$

then $$\vec{M}_o \cdot \vec{e}_{OB} = -636\left(\frac{10}{\sqrt{164}}\right) + 284\left(\frac{8}{\sqrt{164}}\right)$$

$$= -319.23$$

$$\therefore \vec{M}_{OB} = (-249\vec{i} - 199\vec{k})\text{ lb}\cdot\text{in}$$

Example Problem 2.61



Determine the moment of $\vec{F}$ about the axis $\perp$ to ABC and passes through O.

First we need a vector $\perp$ to ABC.

$\vec{AB} \times \vec{AC}$ is $\perp$ to ABC plane

$$\vec{AB} = (-3\vec{j} + 4\vec{k})\, m$$

$$\vec{AC} = (3\vec{i} - 3\vec{j})\, m$$

$$\therefore \vec{e}_{\perp} = \frac{\vec{AB} \times \vec{AC}}{|\vec{AB} \times \vec{AC}|} = \frac{-9\overbrace{\vec{j}\times\vec{i}}^{-\vec{k}} + 9\overbrace{\vec{j}\times\vec{j}}^{0} + 12\overbrace{\vec{k}\times\vec{i}}^{\vec{j}} - 12\overbrace{\vec{k}\times\vec{j}}^{-\vec{i}}}{\sqrt{144+144+81}}$$

$$= \frac{12}{\sqrt{369}}\vec{i} + \frac{12}{\sqrt{369}}\vec{j} + \frac{9}{\sqrt{369}}\vec{k}$$

$$\vec{M}_{O\perp} = (\vec{M}_O \cdot \vec{e}_{\perp})\, \vec{e}_{\perp}$$

$$\vec{M}_O = \vec{r}_{A/O} \times \vec{F} = (3\vec{j})m \times 250\left(\tfrac{-3}{5}\vec{j} + \tfrac{4}{5}\vec{k}\right)N$$
$$= 600\,\vec{i}\ N\cdot m$$

$$\therefore \vec{M}_{O\perp} = 600 \cdot \frac{12}{\sqrt{369}}\left(\frac{12}{\sqrt{369}}\vec{i} + \frac{12}{\sqrt{369}}\vec{j} + \frac{9}{\sqrt{369}}\vec{k}\right) N\cdot m$$