MECH 211 Fall 2006 Test 2 Solutions

1) $r = (0.2 + 0.15\cos\theta)\,m \rightarrow r = 0.330\,m$ at $\theta = 30°$

$\dot{r} = (-0.15\sin\theta\,\dot{\theta})\,m/s \rightarrow \dot{r} = -0.0525\,m/s,\ \theta = 30°, \dot{\theta} = 0.7$

$\ddot{r} = (-0.15\sin\theta\,\ddot{\theta} - 0.15\cos\theta\,\dot{\theta}^2) \rightarrow \ddot{r} = -0.101\,m/s^2,\ \theta = 30°$

$\dot{\theta} = 0.7,\ \ddot{\theta} = 0.5$

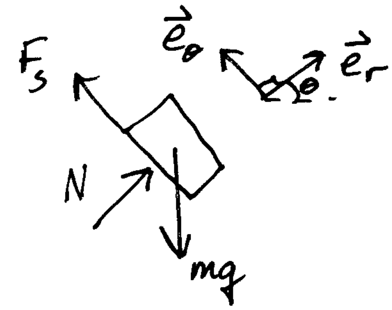
$$\vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$$
$$= (-0.0525\,\vec{e}_r + 0.330 \cdot 0.7\,\vec{e}_\theta)\,m/s$$
$$\vec{v} = (\underbrace{-0.0525}_{v_r}\,\vec{e}_r + \underbrace{0.231}_{v_\theta}\,\vec{e}_\theta)\,m/s$$

$$= (v_r\cos\theta - v_\theta\sin\theta)\vec{i} + (v_r\sin\theta + v_\theta\cos\theta)\vec{j}$$
$$= (\underbrace{-0.161}_{v_x}\,\vec{i} + \underbrace{0.174}_{v_y}\,\vec{j})\,m/s$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$
$$= (-0.101 - 0.330 \cdot 0.7^2)\vec{e}_r + (0.33 \cdot 0.5 + 2 \cdot (-0.0525) \cdot 0.7)\vec{e}_\theta$$
$$\vec{a} = (\underbrace{-0.263}_{a_r}\,\vec{e}_r + \underbrace{0.0915}_{a_\theta}\,\vec{e}_\theta)\,m/s^2$$

$$= (a_r\cos\theta - a_\theta\sin\theta)\vec{i} + (a_r\sin\theta + a_\theta\cos\theta)\vec{j}$$
$$= (\underbrace{-0.274}_{a_x}\,\vec{i} \underbrace{-0.0523}_{a_y}\,\vec{j})\,m/s^2$$

2) FBD:



We are interested in the case where $N=0$ at $\theta=45°$

$$\Sigma F_r = -\frac{\sqrt{2}}{2}mg = m(\ddot{r} - r\dot{\theta}^2)$$

$$\Sigma F_\theta = F_s - \frac{\sqrt{2}}{2}mg = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

To the point we are interested in we know that the block has remained on the surface such that $r = 1.5\text{ ft} = \text{constant}$
$\rightarrow \dot{r} = 0$ and $\ddot{r} = 0$

$$\rightarrow \Sigma F_r = -\frac{\sqrt{2}}{2}mg = -m(1.5)\dot{\theta}^2 \rightarrow \dot{\theta} = \pm 3.896 \frac{\text{rad}}{\text{s}}$$

(+ sign is correct)

When $\theta = 45°$ $\vec{v} = v_\theta \vec{e}_\theta$ b/c $\dot{r} = 0$

$$v_\theta = r\dot{\theta} = 1.5 \cdot 3.896 = 5.844 \text{ ft/s}$$

Energy is conserved

$$\cancel{KE_i}^{\,0} + PE_i + \cancel{W^{NC}}^{\,0} = KE_f + PE_f$$

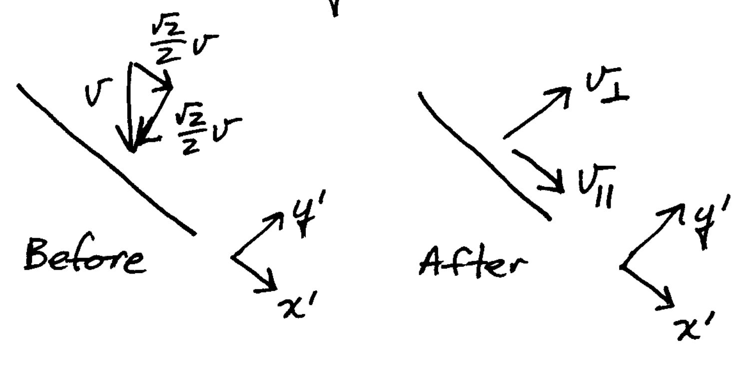
$$\tfrac{1}{2}k(1.5\pi - L_0)^2 = \tfrac{1}{2}\underset{8/32.2}{m}(5.844)^2 + \underset{2}{mg}\frac{\sqrt{2}}{2}(1.5) + \tfrac{1}{2}k\left(1.5\tfrac{3\pi}{4} - L_0\right)^2$$

$$\cancel{L_0^2} - 3\pi L_0 + (1.5\pi)^2 = 1.06 + 2.12 + \cancel{L_0^2} - \frac{9\pi}{4}L_0 + \left(\frac{9\pi}{8}\right)^2$$

$$6.535 = \frac{3\pi}{4}L_0 \rightarrow \boxed{L_0 = 2.77\text{ ft.}}$$

3) Velocity of ball prior to impact from conservation of energy $\rightarrow$ $v = \sqrt{2gh} = 13.9 \text{ ft/s}$

Impact :



Linear momentum conserved in x'-direction

$$\rightarrow \quad m\frac{\sqrt{2}}{2}v = m v_\parallel \rightarrow v_\parallel = \frac{\sqrt{2}}{2}v$$

y' velocities are related to coefficient of restitution

$$e = \frac{v_{sep}}{v_{App}} = \frac{v_\perp}{\frac{\sqrt{2}}{2}v} = 0.8 \rightarrow v_\perp = 0.8\frac{\sqrt{2}}{2}v$$

Transform to x-y ($\uparrow^{y}\rightarrow_{x}$) coordinates:

$$v_x = \frac{\sqrt{2}}{2}v_\perp + \frac{\sqrt{2}}{2}v_\parallel = \frac{1}{2}0.8v + \frac{1}{2}v = 0.9v$$

$$v_y = \frac{\sqrt{2}}{2}v_\perp - \frac{\sqrt{2}}{2}v_\parallel = \frac{1}{2}0.8v - \frac{1}{2}v = -0.1v$$

The ball now can be treated as a projectile in the gravity field.

$$y = \frac{1}{2}a_y t^2 + v_{y0}t + y_0$$

$$0 = -\frac{1}{2}32.2t^2 - 0.1v\,t + 2 \rightarrow t = \frac{1.39 \pm \sqrt{1.39^2 + 8 \cdot 16.1}}{-32.2}$$

$$\rightarrow t = 0.312 \text{ s}$$

$$x = \frac{1}{2}\cancelto{0}{a_x}t^2 + v_{x0}t + x_0$$

$$s = 0.9(13.9)(0.312) - 2 = \boxed{1.9 \text{ ft} = s}$$

Speed at A: $v_y = a_y t + v_{y_0}$

$= -32.2(.312) - 0.1(13.9) = -11.4$ ft/s

$v_x = \cancel{a_x}^{0} t + v_{x_0} = 0.9(13.9) = 12.51$ ft/s

$|v| = \sqrt{v_x^2 + v_y^2} = \underline{\underline{16.9 \text{ ft/s}}}$