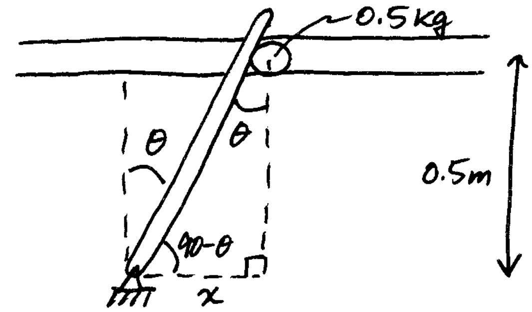
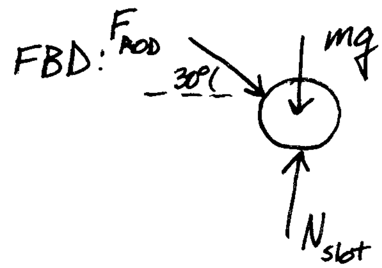
MECH 211 Fall 2003 Test 2

1)





$$\Sigma F_y = N_{slot} - mg - F_{ROD}\sin 30° = m\cancel{a_y}^{\,0}$$ b/c ball moves along slot

$$\therefore\ N_{slot} - mg - \tfrac{1}{2}F_{ROD} = 0$$

$$\Sigma F_x = F_{ROD}\cos 30° = m a_x = m\ddot{x}$$

but

$$x = 0.5\tan\theta$$

$$\dot{x} = 0.5\frac{d}{d\theta}(\tan\theta)\dot{\theta} = 0.5\frac{1}{\cos^2\theta}\dot{\theta}$$

$$\ddot{x} = 0.5\frac{d}{d\theta}\left(\frac{1}{\cos^2\theta}\right)\dot{\theta}\dot{\theta} + 0.5\frac{1}{\cos^2\theta}\ddot{\theta}$$

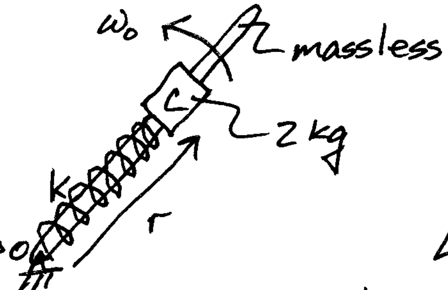
$$\ddot{x} = 0.5\frac{2\sin\theta}{\cos^3\theta}\dot{\theta}^2 + 0.5\frac{1}{\cos^2\theta}\ddot{\theta}$$

$$\left.\begin{aligned}\dot{\theta} &= 2\ \text{rad/s}\\ \ddot{\theta} &= 3\ \text{rad/s}^2\\ \theta &= 30°\end{aligned}\right\}\quad \begin{aligned}\ddot{x} &= 0.5\cdot 2\,\frac{1/2\cdot 2^2}{3/4\cdot\sqrt{3}/2} + 0.5\,\frac{1}{3/4}\cdot 3\\ \ddot{x} &= 5.08\ \text{m/s}^2\end{aligned}$$

$$\therefore\ \frac{\sqrt{3}}{2}F_{ROD} = 0.5(5.08) \rightarrow \boxed{F_{ROD} = 2.93\,N}$$

$$\rightarrow N_{slot} = 0.5(9.81) + \tfrac{1}{2}(2.93) \rightarrow \boxed{N_{slot} = 6.37\,N}$$

2) Rotates freely
Spring force acts through O. No $M_z^o \rightarrow h_z^o$ conserved.



$k = 40\ N/m$

$L_0 = 0.8\ m$

$\omega_0 = 6\ rad/s$ when $r = 0.2\ m$

Angular momentum about O is conserved

$$\rightarrow \quad m R_i v_{\theta i} = m R_f v_{\theta f} \qquad v_\theta = R\dot{\theta}$$

$$m(0.2)(0.2\cdot 6) = m(0.8)\, v_{\theta f} \rightarrow \boxed{v_{\theta f} = 0.3\ \tfrac{m}{s} \;\rightarrow\; \dot{\theta}_f = \frac{v_{\theta f}}{R_f} = 0.375\ \tfrac{rad}{s}}$$

Energy is conserved

$$\rightarrow KE_i + PE_i + \cancel{W_{NC}}^{\,0} = KE_f + PE_f$$

$$\frac{1}{2}m(\cancel{v_{ri}^2}^{\,0} + v_{\theta i}^2) + \frac{1}{2}k\delta_i^2 = \frac{1}{2}m(v_{rf}^2 + v_{\theta f}^2) + \frac{1}{2}k\cancel{\delta_f^2}^{\,0}$$

$$\frac{1}{2}\underbrace{m}_{2}(0.2\cdot 6)^2 + \frac{1}{2}\underbrace{k}_{40}(0.6)^2 = \frac{1}{2}\underbrace{m}_{2}(v_{rf}^2 + 0.3^2)$$

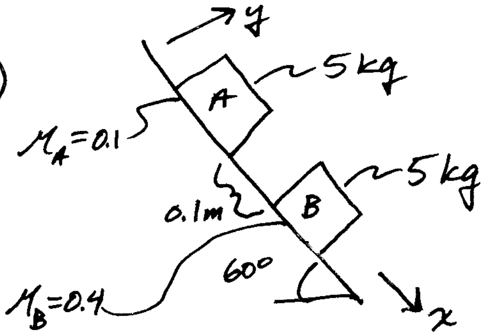
$$\therefore \quad v_{rf}^2 = (0.2\cdot 6)^2 + 20(.6)^2 - 0.3^2 = 8.55$$

$$\rightarrow \quad v_{rf} = \pm 2.92\ \tfrac{m}{s}$$ (+ if the spring is in the process of extending; − if it is contracting)

$$\therefore \quad \vec{v}_f = v_{rf}\,\vec{e}_r + v_{\theta f}\,\vec{e}_\theta$$

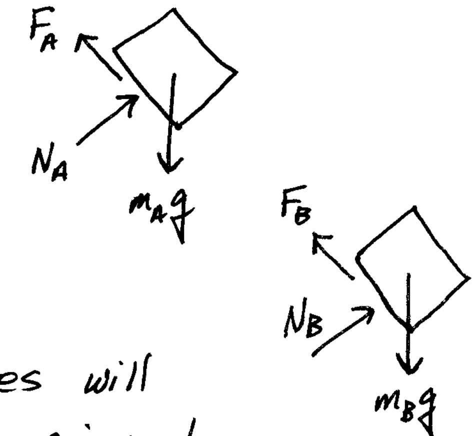
$$\boxed{\vec{v}_f = \pm 2.92\ \tfrac{m}{s}\,\vec{e}_r + 0.3\ \tfrac{m}{s}\,\vec{e}_\theta} \rightarrow \boxed{|v_f| = 2.94\ \tfrac{m}{s}}$$

3)



$e = 0.8$

FBDs: (prior to collision)



We need to find when the crates will collide and how fast they are going when they do collide.

For either crate: $\Sigma F_y = N - mg\cos 60° = m\cancel{a_y}^{\,0}$

$$\therefore N = \tfrac{1}{2} mg$$

$$\Sigma F_x = mg\sin 60° - \underbrace{F}_{\mu N} = m a_x$$

$$\therefore \frac{\sqrt{3}}{2} mg - \frac{1}{2}\mu mg = m a_x$$

$$a_x = \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\mu\right) g$$

$$\therefore \quad a_x^A = \left(\frac{\sqrt{3}}{2} - \frac{1}{2}(0.1)\right) 9.81 = 8.01 \text{ m/s}^2$$
$$a_x^B = \left(\frac{\sqrt{3}}{2} - \frac{1}{2}(0.4)\right) 9.81 = 6.53 \text{ m/s}^2$$

Both crates have <u>constant</u> accelerations in the x-direction so we can find the distance traveled by each crate using

$d = \frac{1}{2}at^2 + v_0 t$, $v_0 = 0$ b/c both crates start from rest.

$\therefore \quad d_A = \frac{1}{2}(8.01)t^2$ and $d_B = \frac{1}{2}(6.53)t^2$

The crates collide when $d_A = d_B + 0.1$

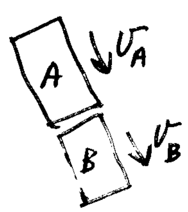
$\rightarrow \quad \frac{1}{2}(8.01)t^2 = \frac{1}{2}(6.53)t^2 + 0.1$

$\rightarrow \boxed{t = 0.3676\ s}$

$\therefore \quad v_A = a_A t = 8.01 \cdot .3676 = 2.945 \frac{m}{s}$

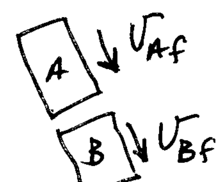
$v_B = a_B t = 6.53 \cdot .3676 = 2.400 \frac{m}{s}$

For the instant of the collision x-mom of the system is conserved



before

$v_{APP} = v_A - v_B$



after

$v_{sep} = v_{Bf} - v_{Af}$

$$m_A v_A + m_B v_B = m_A v_{Af} + m_B v_{Bf}$$

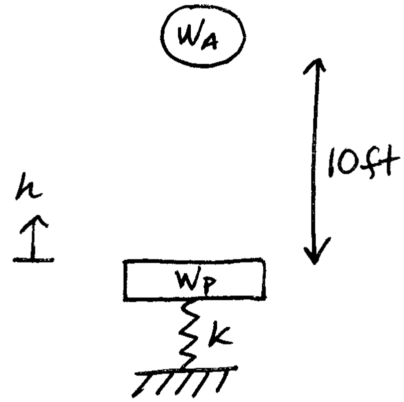
$$\rightarrow \quad v_{Af} + v_{Bf} = v_A + v_B = 5.345$$

$$e = 0.8 = \frac{v_{sep}}{v_{APP}} = \frac{v_{Bf} - v_{Af}}{0.545} \rightarrow v_{Bf} - v_{Af} = 0.436$$

$\therefore$

$$\boxed{\begin{array}{l} v_{Bf} = 2.89\ m/s \searrow \\ v_{Af} = 2.45\ m/s \searrow \end{array}}$$

4)



$W_A = m_A g = 8 \text{ lb}$

$W_P = m_P g = 16 \text{ lb}$

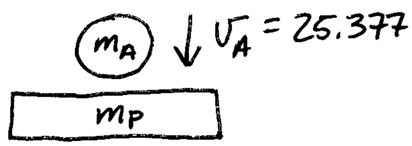
$k = 3 \text{ lb/in} = 36 \text{ lb/ft}$

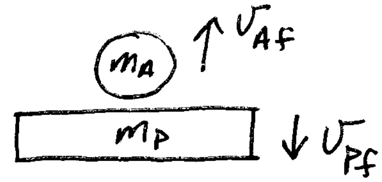
Prior to collision, energy of ball is conserved.

$$\cancel{KE_i}^{\,0} + PE_i + \cancel{W_{NC}}^{\,0} = KE_f + \cancel{PE_f}^{\,0} \rightarrow m_A g h_i = \frac{1}{2} m_A v_A^2$$

$$\rightarrow v_A = \sqrt{2 \cdot 32.2 \cdot 10} = 25.377 \text{ ft/s}$$

Collision:





before: $v_{APP} = 25.377$ after: $v_{sep} = v_{Pf} + v_{Af}$

$$e = \frac{v_{sep}}{v_{APP}} \rightarrow 1 = \frac{v_{Pf} + v_{Af}}{25.377} \rightarrow v_{Pf} + v_{Af} = 25.377 \quad (1)$$

Cons. y-momentum $+\uparrow$: $-m_A v_A = m_A v_{Af} - m_P v_{Pf}$

$$-\frac{8}{\cancel{g}}(25.377) = \frac{8}{\cancel{g}} v_{Af} - \frac{16}{\cancel{g}} v_{Pf}$$

$$v_{Af} - 2 v_{Pf} = -25.377 \quad (2)$$

$$\therefore (1) \,\&\, (2) \rightarrow v_{Pf} = \frac{2 \cdot 25.377}{3} = 16.918 \text{ ft/s}$$

$$\hookrightarrow v_{Af} = 25.377 - 16.918 = 8.459 \text{ ft/s}$$

After collision, energy of ball is conserved

$$KE_i + \cancel{PE_i}^{\,0} + \cancel{W_{NC}}^{\,0} = \cancel{KE_f}^{\,0} + PE_f \rightarrow \frac{1}{2} \cancel{m_A} (8.459)^2 = \cancel{m_A} g h_{max}$$

$$\therefore \boxed{h_{max} = \frac{8.459^2}{2 \cdot 32.2} = 1.11 \text{ ft}}$$

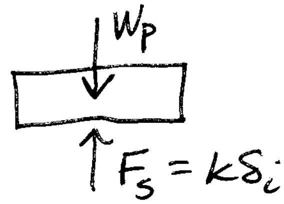
After collision, energy of plate & spring are conserved

$$KE_i + PE_i + \cancel{W_{NC}}^{\,0} = \cancel{KE_f}^{\,0} + PE_f$$

$$\frac{1}{2} m_p (16.918)^2 + \frac{1}{2} k \delta_i^2 = \frac{1}{2} k \delta_{max}^2 - m_p g (\delta_{max} - \delta_i)$$

2 unknowns δ_i & δ_{max}

δ_i can be determined from an FBD of the plate before the collision.



$$\Sigma F_y = k\delta_i - W_p = m_p \cancel{a_y}^{\,0}$$

$$\rightarrow \delta_i = \frac{W_p}{k} = \frac{16}{36} = 0.444 \text{ ft.}$$

$$\therefore \frac{1}{2} \frac{16}{32.2} (16.918)^2 + \frac{1}{2} \cdot 36 \cdot \left(\frac{4}{9}\right)^2 = \frac{1}{2} \cdot 36 \cdot \delta_{max}^2 - \frac{16}{32.2} \cdot 32.2 \left(\delta_{max} - \frac{4}{9}\right)$$

$$\rightarrow \boxed{18\,\delta_{max}^2 - 16\,\delta_{max} - 67.555 = 0}$$

Solution :

$$\delta_{max} = \frac{16 \pm \sqrt{16^2 + 4(67.555 \cdot 18)}}{2 \cdot 18}$$

$$= \frac{16 \pm 71.55}{36} \quad \text{use } + \rightarrow \frac{16 + 71.55}{36}$$

$$\boxed{\delta_{max} = 2.43 \text{ ft}}$$