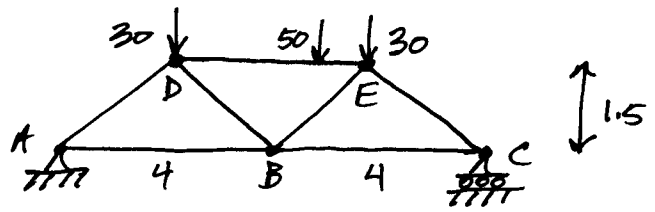
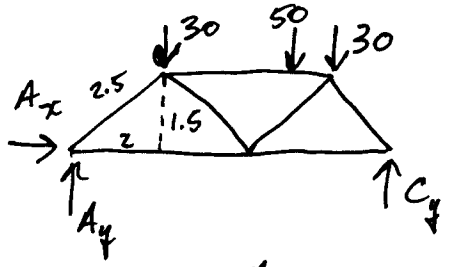
Fall 2006 Test 1 Solutions

1)



a) Entire Structure



$$\Sigma F_x = \boxed{A_x = 0}$$

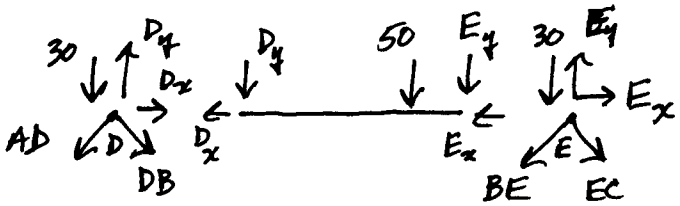
$$\Sigma F_y = A_y + C_y - 30 - 30 - 50 = 0$$

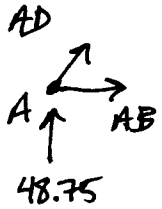
$$\Sigma M_z^A = 8C_y - 6(30) - 5(50) - 2(30) = 0$$

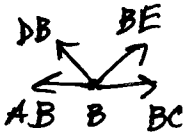
$$\rightarrow \boxed{C_y = 61.25\ kN}$$

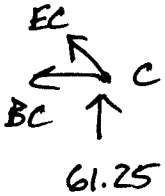
$$\rightarrow \boxed{A_y = 48.75\ kN}$$

Joints + DE









A) $\Sigma F_y = 48.75 + \frac{3}{5}AD = 0 \rightarrow \boxed{AD = -81.25\ kN\ (c)}$

$\Sigma F_x = AB + \frac{4}{5}AD = 0 \rightarrow \boxed{AB = 65\ kN\ (t)}$

DE) $\Sigma F_x = -D_x - E_x = 0$

$\Sigma F_y = -50 - D_y - E_y = 0 \rightarrow D_y = -12.5\ kN$

$\Sigma M_z^D = -50(3) - E_y(4) = 0 \rightarrow E_y = -37.5\ kN$

D) $\Sigma F_y = -30 - \frac{3}{5}AD - \frac{3}{5}DB + D_y = 0$

$-30 + 48.75 - \frac{3}{5}DB - 12.5 = 0 \rightarrow$ **$DB = 10.42\ kN\ (t)$**

$\Sigma F_x = \frac{-4}{5}AD + \frac{4}{5}DB + D_x = 0$

$65 + 8.33 + D_x = 0 \rightarrow D_x = -73.33\ kN$

from ΣF_x for DE $\rightarrow E_x = 73.33\ kN$

B) $\Sigma F_y = \frac{3}{5}DB + \frac{3}{5}BE = 0 \rightarrow$ **$BE = -10.42\ kN\ (c)$**

$\Sigma F_x = \frac{-4}{5}DB + \frac{4}{5}BE - AB + BC = 0$

$\frac{-8}{5}(10.42) - 65 + BC = 0 \rightarrow$ **$BC = 81.67\ kN\ (t)$**

E) $\Sigma F_x = \frac{-4}{5}BE + E_x + \frac{4}{5}EC = 0$

$\frac{-4}{5}(-10.42) + 73.33 + \frac{4}{5}EC = 0 \rightarrow$ **$EC = -102.08\ kN\ (c)$**

check: $\Sigma F_y = -30 + E_y - \frac{3}{5}BE - \frac{3}{5}EC$

$= -30 - 37.5 - \frac{3}{5}(-10.42) - \frac{3}{5}(-102.08)$

$= 0$ ✓

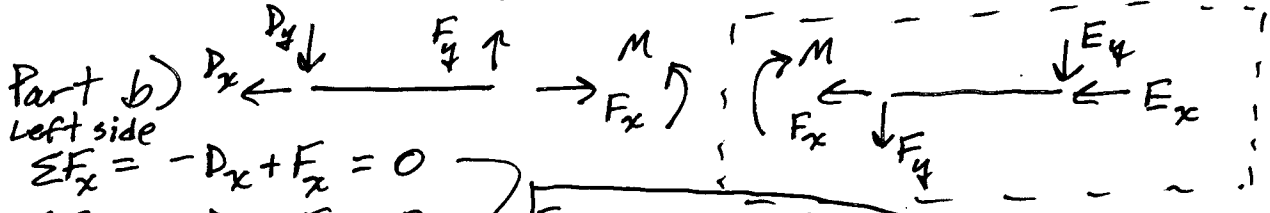
check C) $\Sigma F_x = -\frac{4}{5}EC - BC$

$= \frac{-4}{5}(-102.08) - 81.67 = 0$ ✓

$\Sigma F_y = \frac{3}{5}EC + 61.25$

$= \frac{3}{5}(-102.08) + 61.25 = 0$ ✓

Part b) Left side



$\Sigma F_x = -D_x + F_x = 0$

$\Sigma F_y = -D_y + F_y = 0$

$\Sigma M_z^{4/2} = M + 2D_y = 0$

$F_x = -73.33\ kN$

$F_y = -12.5\ kN$

$M = 25\ kN \cdot m$

2) $\vec{M}_{AC} = (\vec{M}_C \cdot \vec{e}_{AC})\,\vec{e}_{AC}$

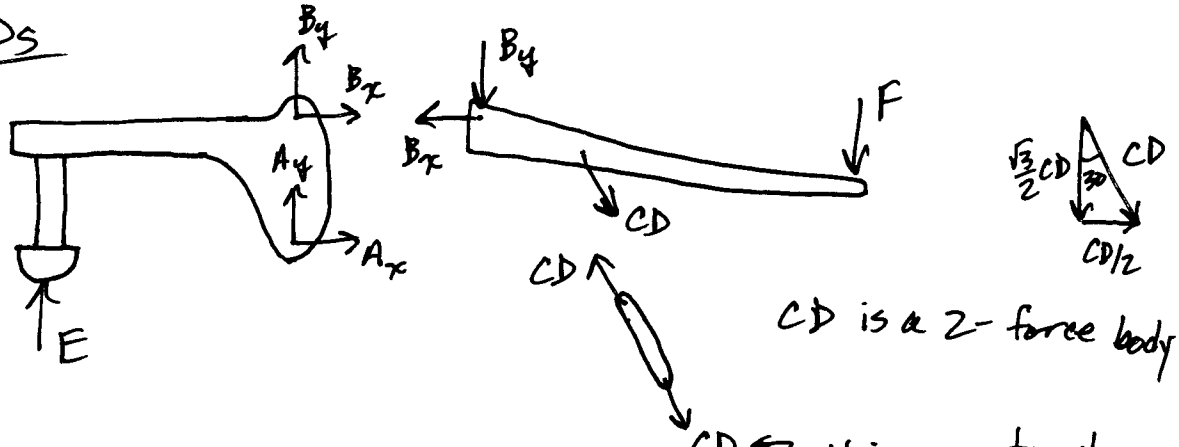
$$\vec{M}_C = \vec{r}_{B/C} \times \vec{F} = -2\vec{k} \times (4\vec{i} + 12\vec{j} - 3\vec{k})$$
$$= -8\underbrace{\vec{k}\times\vec{i}}_{\vec{j}} - 24\underbrace{\vec{k}\times\vec{j}}_{-\vec{i}}$$
$$= 24\vec{i} - 8\vec{j} \text{ lb·ft}$$

$$\vec{e}_{AC} = \frac{4\vec{i} + 3\vec{j}}{\sqrt{16+9}} = \frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}$$

$$\vec{M}_C \cdot \vec{e}_{AC} = 24\cdot\frac{4}{5} - 8\cdot\frac{3}{5} = 14.4 \text{ lb·ft}$$

$$\rightarrow \boxed{\vec{M}_{AC} = 14.4\left(\frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}\right) = 11.52\vec{i} + 8.64\vec{j}}$$

3) FBDs



CD is a 2-force body

CD ← this reacts w/ ground

Handle: $\Sigma M_z^B = -2aF + \frac{a}{2}\frac{CD}{2} - \frac{a}{2}\frac{\sqrt{3}}{2}CD = 0$

$$\rightarrow CD = -\frac{8}{\sqrt{3}-1}F = -10.9282F$$

$$\Sigma F_x = -B_x + \frac{CD}{2} = 0 \rightarrow B_x = -5.4641F$$

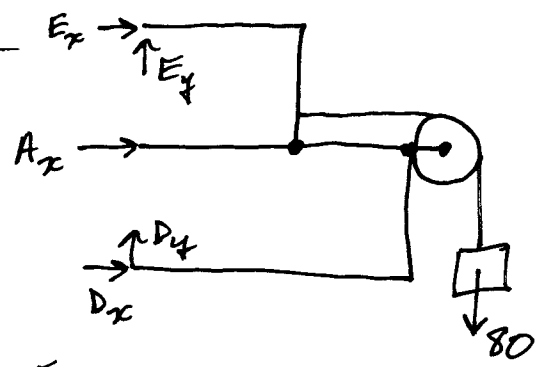
we will not need ΣF_y to determine E

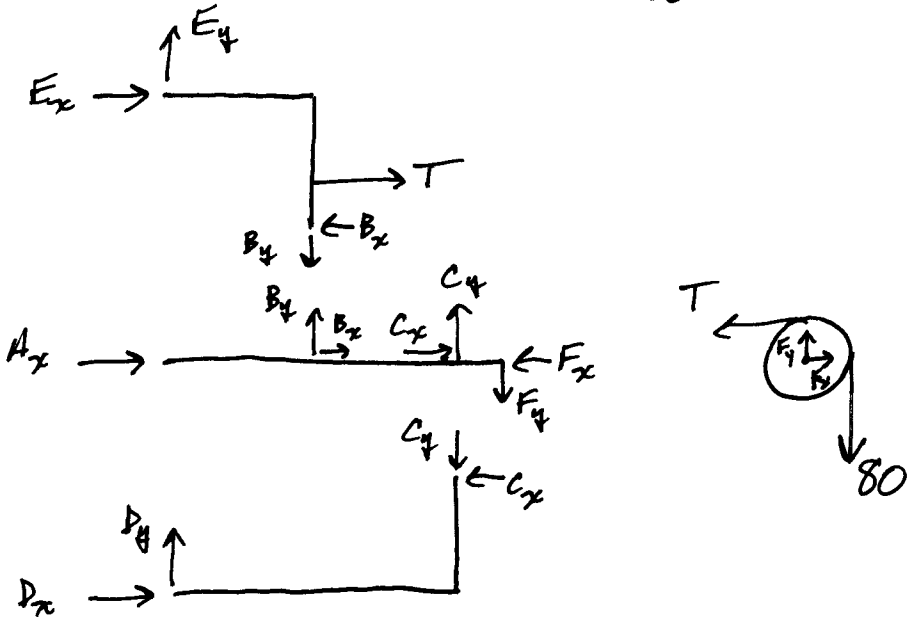
Clamp: $\Sigma M_z^A = -B_x \cdot a - E \cdot 1.5a = 0$

$\rightarrow E = \frac{-B_x}{1.5} = 3.64F$

$$\boxed{E = 3.64F}$$

4) FBDs





Pulley: $\Sigma M_z^F = -80 \cdot 1 + T \cdot 1 = 0 \rightarrow T = 80\ lb.$

$\Sigma F_y = 0 = -80 + F_y = 0 \rightarrow F_y = 80\ lb.$

$\Sigma F_x = 0 = -T + F_x = 0 \rightarrow F_x = 80\ lb.$

Bar AF: $\Sigma M_z^B = 0 = C_y \cdot 3 - F_y \cdot 4 = 0$

$$\rightarrow \boxed{C_y = \frac{4}{3}F_y = 106.67\ lb.}$$

$$\Sigma F_y = B_y + C_y - F_y = 0 \rightarrow \boxed{B_y = -26.67 \text{ lb.}}$$

Right now ΣF_x for Bar AF will not help us.

Bar EB: $\Sigma M_z^E = -B_y \cdot 3 - B_x \cdot 4 + T \cdot 3 = 0$

$$26.67 \cdot 3 - 4B_x + 80 \cdot 3 = 0$$

$$\rightarrow \boxed{B_x = 80 \text{ lb.}}$$

$$\Sigma F_y = E_y - B_y = 0 \rightarrow \boxed{E_y = -26.67 \text{ lb.}}$$

$$\Sigma F_x = E_x + \underset{\text{in } 80}{T} - \underset{\text{in } 80}{B_x} = 0 \rightarrow \boxed{E_x = 0}$$

Bar CD: $\Sigma M_z^D = -C_y \cdot 6 + C_x \cdot 4 = 0$

$$\rightarrow \boxed{C_x = 160 \text{ lb.}}$$

$$\Sigma F_x = D_x - C_x = 0 \rightarrow \boxed{D_x = 160 \text{ lb.}}$$

$$\Sigma F_y = D_y - C_y = 0 \rightarrow \boxed{D_y = 106.67 \text{ lb.}}$$

Bar AF: $\Sigma F_x = A_x + \underset{\text{in } 80}{B_x} + \underset{\text{in } 160}{C_x} - \underset{\text{in } 80}{F_x} = 0$

$$\rightarrow \boxed{A_x = -160 \text{ lb.}}$$