Fall 2004 Test 1 Solutions

1) a) $\vec{M}_o = \vec{r}_{OF} \times \vec{F}$

$$\vec{r}_{OF} = (6\vec{i} + 14\vec{j} + 10\vec{k})\ \text{in.}$$

$$\vec{F} = 20(\cos\theta_x \vec{i} + \cos\theta_y \vec{j} + \cos\theta_z \vec{k})$$

$$\theta_x = 60^\circ \quad \theta_y = 120^\circ \quad \theta_z = 45^\circ$$

$$= 20\left(\tfrac{1}{2}\vec{i} - \tfrac{1}{2}\vec{j} + \tfrac{\sqrt{2}}{2}\vec{k}\right)\ \text{lbs.}$$

$$\vec{M}_o = (6\vec{i} + 14\vec{j} + 10\vec{k}) \times 20\left(\tfrac{1}{2}\vec{i} - \tfrac{1}{2}\vec{j} + \tfrac{\sqrt{2}}{2}\vec{k}\right)$$

$$= -60\vec{k} - 60\sqrt{2}\vec{j} - 140\vec{k} + 140\sqrt{2}\vec{i} + 100\vec{j} + 100\vec{i}$$

$$\boxed{\vec{M}_o = (298\vec{i} + 15.1\vec{j} - 200\vec{k})\ \text{lb}\cdot\text{in}}$$

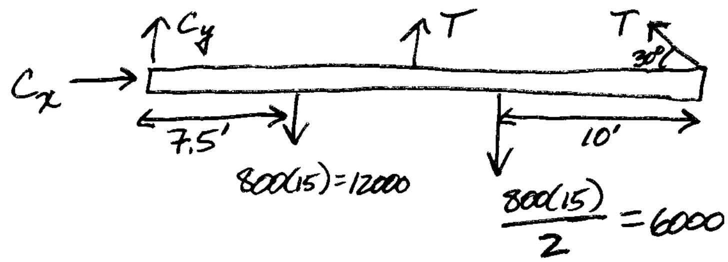
b) $\vec{M}_{AXIS\,OA} = \left[(\vec{r}_{OF} \times \vec{F}) \cdot \vec{e}_{OA}\right] \vec{e}_{OA}$

$$\vec{e}_{OA} = \frac{6\vec{i} + 8\vec{j}}{\sqrt{36+64}} = 0.6\vec{i} + 0.8\vec{j}$$

$$(\vec{r}_{OF} \times \vec{F}) \cdot \vec{e}_{OA} = \vec{M}_o \cdot \vec{e}_{OA} = 298(0.6) + 15.1(0.8) = 190.88$$

$$(\vec{M}_o \cdot \vec{e}_{OA})\vec{e}_{OA} = \boxed{\vec{M}_{AXIS\,OA} = 114.5\vec{i} + 152.7\vec{j}\ \text{lb}\cdot\text{in}}$$

2) FBD



Equilibrium

$$\Sigma F_x = C_x - \frac{\sqrt{3}}{2} T = 0$$

$$\Sigma F_y = C_y + T + \frac{1}{2} T - 12000 - 6000 = 0$$

$$\Sigma M_z^c = -12000(7.5) + 15T - 6000(20) + \frac{1}{2} T(30) = 0$$

$$30T = 210000$$

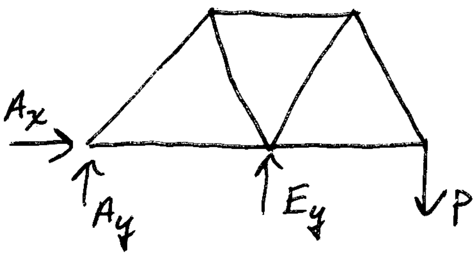
$$T = 7000$$

$$\rightarrow \quad C_x = \frac{\sqrt{3}}{2} T = 6062$$

$$\rightarrow \quad C_y = 18000 - \frac{3}{2} T = 7500$$

$$\boxed{\begin{array}{lll} C_x = 6062 \text{ lbs} & \rightarrow \\ C_y = 7500 \text{ lbs} & \uparrow \\ T = 7000 \text{ lbs} & \end{array}}$$

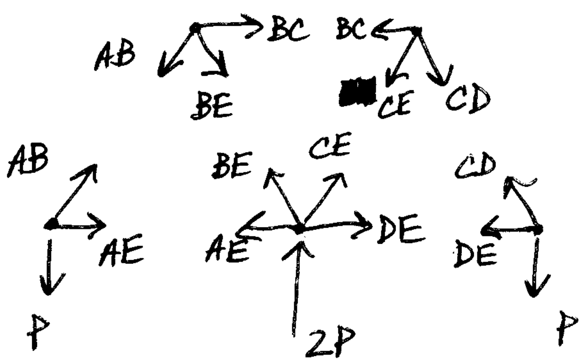
3) FBDs



$\Sigma F_x = \boxed{A_x = 0}$

$\Sigma M_z^E = -4P - 4A_y = 0$
$\rightarrow A_y = -P$

$\Sigma F_y = A_y + E_y - P = 0$
$\rightarrow E_y = 2P$



You could notice that everything is symmetric giving $AE = DE$, $CE = BE$, and $AB = CD$ to help simplify.

Equilibrium at joints

A) $\Sigma F_y = \frac{\sqrt{3}}{2}AB - P = 0 \rightarrow \boxed{AB = 2P/\sqrt{3}}$
$\Sigma F_x = AE + \frac{1}{2}AB = 0 \rightarrow \boxed{AE = -P/\sqrt{3}}$

D) $\Sigma F_y = \frac{\sqrt{3}}{2}CD - P = 0 \rightarrow \boxed{CD = \frac{2}{\sqrt{3}}P}$
$\Sigma F_x = -DE - \frac{1}{2}CD = 0 \rightarrow \boxed{DE = \frac{-1}{\sqrt{3}}P}$

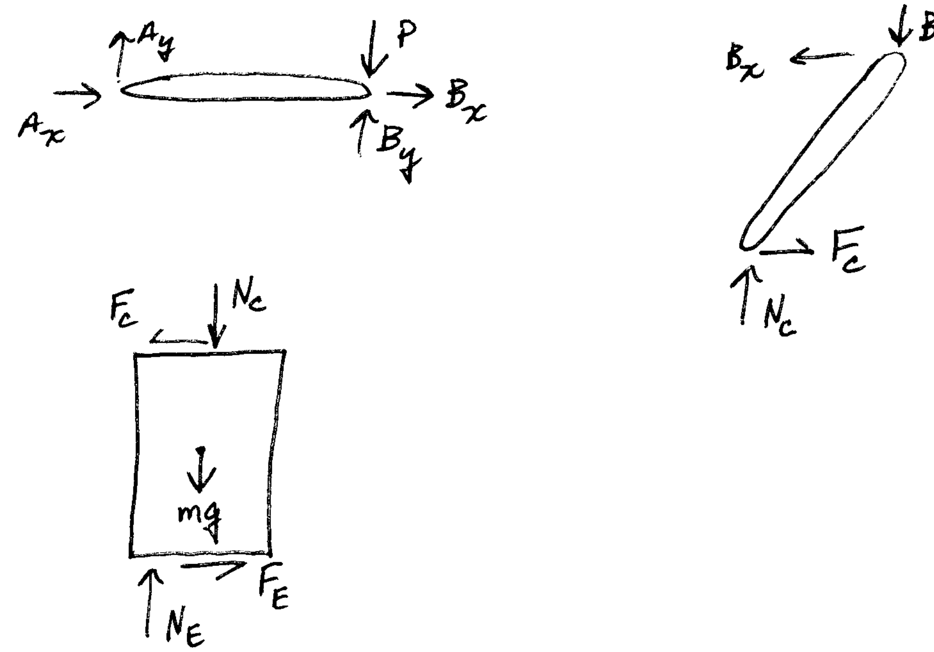
B) $\Sigma F_y = -\frac{\sqrt{3}}{2}AB - \frac{\sqrt{3}}{2}BE = 0 \rightarrow \boxed{BE = \frac{-2}{\sqrt{3}}P}$
$\Sigma F_x = -\frac{1}{2}AB + \frac{1}{2}BE + BC = 0 \rightarrow \boxed{BC = \frac{2}{\sqrt{3}}P}$

C) $\Sigma F_y = -\frac{\sqrt{3}}{2}CE - \frac{\sqrt{3}}{2}CD = 0 \rightarrow \boxed{CE = \frac{-2}{\sqrt{3}}P}$
$\Sigma F_x = \frac{1}{2}CD - \frac{1}{2}CE - BC$
$= \frac{1}{\sqrt{3}}P + \frac{1}{\sqrt{3}}P - \frac{2}{\sqrt{3}}P = 0$ ✓

Check E) $\Sigma F_x = AE - DE - \frac{1}{2}BE + \frac{1}{2}CE$
$= -\frac{P}{\sqrt{3}} + \frac{P}{\sqrt{3}} + \frac{P}{\sqrt{3}} - \frac{P}{\sqrt{3}} \stackrel{\checkmark}{=} 0$
$\Sigma F_y = 2P + \frac{\sqrt{3}}{2}BE + \frac{\sqrt{3}}{2}CE = 2P - P - P \stackrel{\checkmark}{=} 0$

$AB = CD = BC = \frac{2}{\sqrt{3}}P$ (tension) $\leq 8kN \rightarrow P \leq 6.9kN$
$AE = DE = \frac{1}{\sqrt{3}}P$ (compression) $\leq 6kN \rightarrow P \leq 10.4kN$
$CE = BE = \frac{2}{\sqrt{3}}P$ (compression) $\leq 6kN \rightarrow \boxed{P \leq 5.2kN}$
CE & BE are critical, $\boxed{\text{max } P = 5.2kN.}$

4) FBDs



Equilibrium : Bar AB : $\sum M_z^B = A_y(100) = 0 \rightarrow A_y = 0$

$\sum F_y = B_y - P = 0 \rightarrow B_y = P$

Bar BC : $\sum M_z^C = -B_y(100) + B_x(\overset{\tan 60^\circ}{\sqrt{3}} \cdot 100) = 0$

$\rightarrow B_x = P/\sqrt{3}$

$\left.\begin{array}{l} \sum F_x = F_c - B_x = 0 \rightarrow F_c = P/\sqrt{3} \\ \sum F_y = N_c - B_y = 0 \rightarrow N_c = P \end{array}\right\} F_c \leq \mu_c N_c$

$\frac{P}{\sqrt{3}} \leq 0.6P$

$0.58P \leq 0.6P$ ✓

$\rightarrow$ no sliding at C ever

Block D : $\left.\begin{array}{l} \sum F_x = F_E - F_c = 0 \rightarrow F_E = P/\sqrt{3} \\ \sum F_y = N_E - mg - N_c = 0 \rightarrow N_E = P + 490.5 \end{array}\right\} F_E \leq \mu_E N_E$

$\frac{P}{\sqrt{3}} \leq 0.3(P + 490.5)$

$\rightarrow P \leq 530.6\ N$ or it will slide

at imp. tipping

$\rightarrow \sum M_z^E = -mg(\cancel{50}) - N_c(\cancel{50}) + F_c(\overset{4}{\cancel{200}}) = 0$

$-490.5 - P + \frac{4}{\sqrt{3}}P = 0$

$P = 374.6\ N$ at impending tipping

$\therefore$ $\boxed{\text{The block will tip about point E when } P = 374.6\ N}$