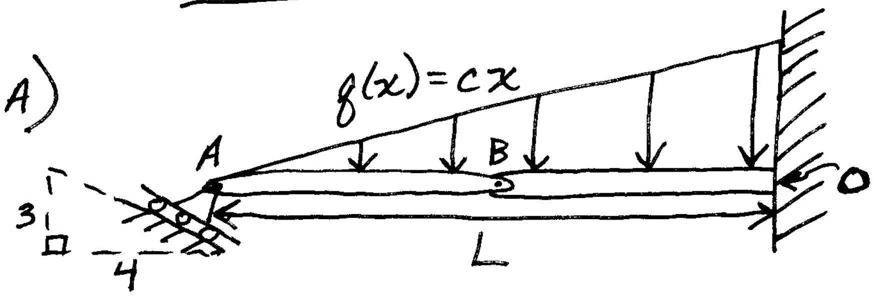
Problems from 2001 Test

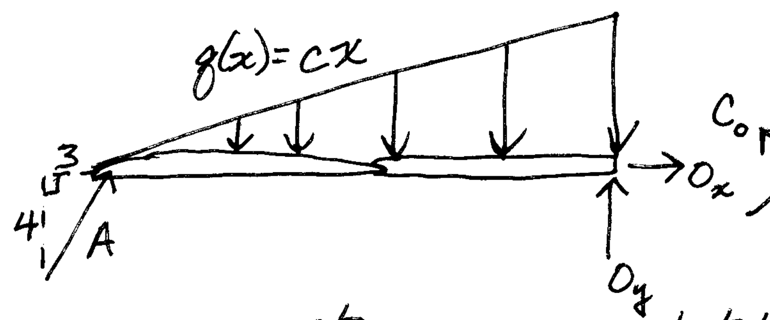
A)



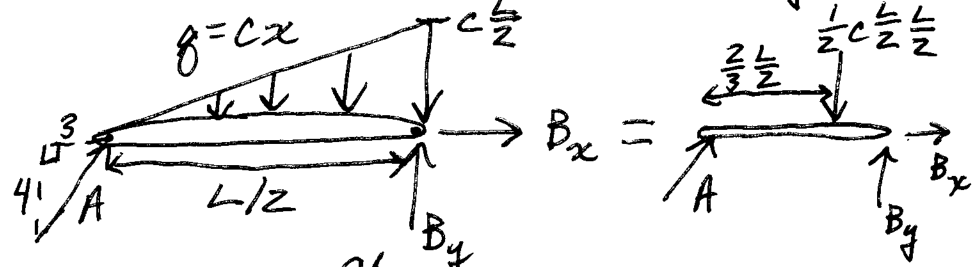
Reactions at O and A?

FBD's

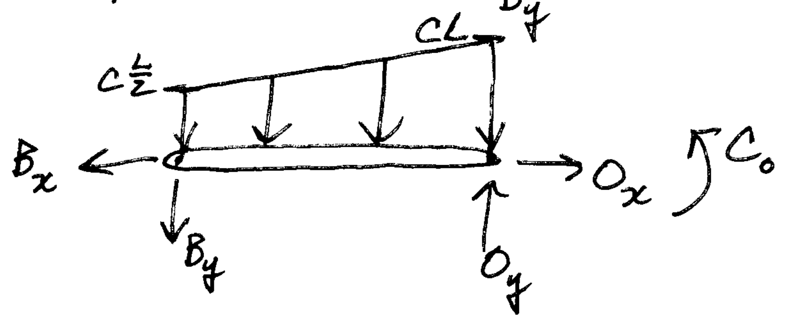
Entire structure:



AB :



BO :



Analysis : Entire structure → 4 unknowns A, O_x, O_y, C_o
3 equations

AB : 3 unkowns A, B_x, B_y
3 equations

BO : 5 unknowns B_x, B_y, O_x, O_y, C_o
3 equations

Analyze AB first: $\Sigma F_x = \frac{3}{5}A + B_x = 0$

$$\Sigma F_y = \frac{4}{5}A - \frac{1}{8}cL^2 + B_y = 0$$

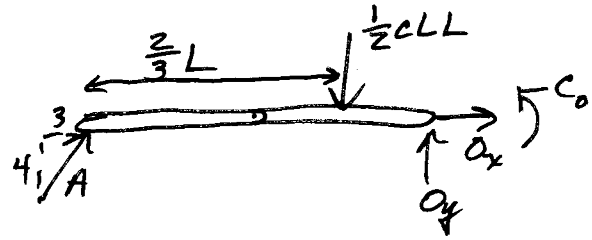
$$\Sigma M_z^B = \underbrace{-\frac{4}{5}A}_{A_y}\frac{L}{2} + \frac{1}{8}cL^2\frac{L}{6} = 0$$

$$\therefore \boxed{A = \frac{5}{96}cL^2}$$

$$\rightarrow B_x = -\frac{3}{96}cL^2 , \quad B_y = \frac{8}{96}cL^2$$

Now analyze either BO or the entire structure.

Entire structure :



$$\Sigma F_x = \frac{3}{5}A + O_x = 0 \rightarrow O_x = -\frac{3}{5}A = \frac{-3}{96}cL^2$$

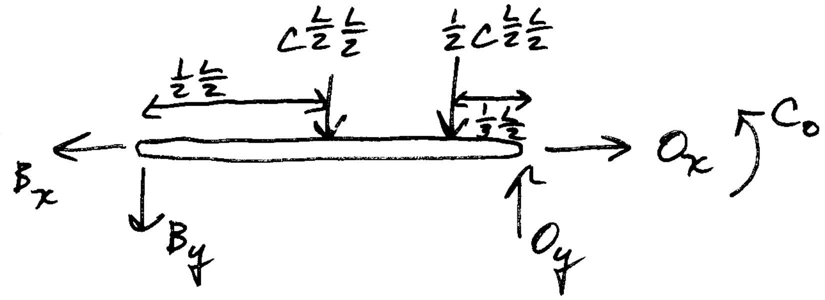
$$\Sigma F_y = \frac{4}{5}A - \frac{1}{2}cL^2 + O_y \rightarrow O_y = \frac{1}{2}cL^2 - \frac{4}{96}cL^2 = \frac{44}{96}cL^2$$

$$\Sigma M_z^o = C_o + \frac{1}{2}cL^2\frac{L}{3} - \frac{4}{5}AL = 0$$

$$C_o = \frac{4}{5}\frac{5}{96}cL^2L - \frac{1}{6}cL^3 = \frac{-12}{96}cL^3$$

Now we can check this solution by analyzing BO.

BO :



$$\Sigma F_x = -B_x + O_x = +\frac{3}{96}cL^2 - \frac{3}{96}cL^2 = 0 \checkmark$$

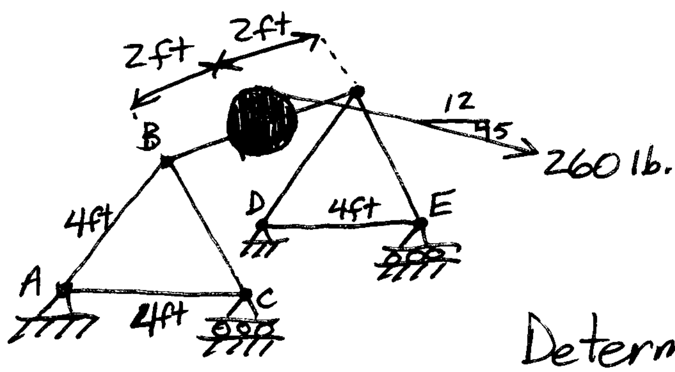
$$\Sigma F_y = -B_y - \frac{1}{4}cL^2 - \frac{1}{8}cL^2 + O_y = \frac{-8}{96}cL^2 - \frac{24}{96}cL^2 - \frac{12}{96}cL^2 + \frac{44}{96}cL^2 = 0 \checkmark$$

$$\Sigma M_z^o = C_o + B_y\frac{L}{2} + \frac{1}{4}cL^2\frac{L}{4} + \frac{1}{8}cL^2\frac{L}{6} = \frac{-12}{96}cL^3 + \frac{4}{96}cL^3 + \frac{6}{96}cL^3 + \frac{2}{96}cL^3 = 0 \checkmark$$

$\therefore$

$$\boxed{\begin{array}{l} A = \frac{5}{96}cL^2 \text{ or } A_x = \frac{3}{96}cL^2,\ A_y = \frac{4}{96}cL^2 \\ O_x = -\frac{3}{96}cL^2,\quad O_y = \frac{44}{96}cL^2 \\ C_o = \frac{-12}{96}cL^3 \end{array}}$$

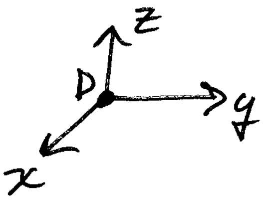
B)



Radius of spool = 1 ft.
Weight of spool = 900 lb.

Determine the moment about the axis ~~[illegible]~~ through C and D due to both the weight of the spool and the 260 lb. tension.

Choose a coordinate system $\rightarrow$

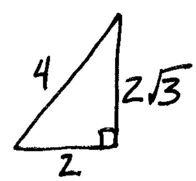


$$\vec{M}_{CD} = (\vec{M}_D \cdot \vec{e}_{CD})\,\vec{e}_{CD}$$

$$\vec{M}_D = \vec{r}_{T/D} \times \vec{T} + \vec{r}_{W/D} \times \vec{W}$$

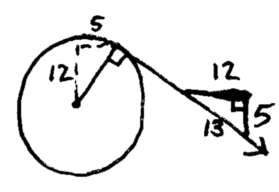
$$\vec{e}_{CD} = \frac{4\vec{i} + 4\vec{j}}{\sqrt{16+16}} = \frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j}$$

$$\vec{W} = -900\,\vec{k}$$



$$\vec{r}_{W/D} = 2\vec{i} + 2\vec{j} + 2\sqrt{3}\,\vec{k}$$

$$\vec{r}_{T/D} = 2\vec{i} + \left(2 + \tfrac{5}{13}\right)\vec{j} + \left(2\sqrt{3} + \tfrac{12}{13}\right)\vec{k}$$



$$\vec{T} = 260\left(\tfrac{12}{13}\vec{j} - \tfrac{5}{13}\vec{k}\right)$$
$$= 240\,\vec{j} - 100\,\vec{k}$$

$$\therefore\ \vec{M}_D = 480\,\overbrace{\vec{i}\times\vec{j}}^{\vec{k}} - 200\,\overbrace{\vec{i}\times\vec{k}}^{-\vec{j}} - 100\left(2+\tfrac{5}{13}\right)\overbrace{\vec{j}\times\vec{k}}^{\vec{i}} + 240\left(2\sqrt{3}+\tfrac{12}{13}\right)\underbrace{\vec{k}\times\vec{j}}_{-\vec{i}} \quad \Big\}\ \vec{r}_{T/D}\times\vec{T}$$

$$+ -1800\,\underbrace{\vec{i}\times\vec{k}}_{-\vec{j}} - 1800\,\underbrace{\vec{j}\times\vec{k}}_{\vec{i}} \quad \Big\}\ \vec{r}_{W/D}\times\vec{W}$$

Then $\vec{M}_D \cdot \vec{e}_{CD} = \frac{\sqrt{2}}{2}\left[-100\left(2+\tfrac{5}{13}\right) - 240\left(2\sqrt{3}+\tfrac{12}{13}\right) - 1800\right]$

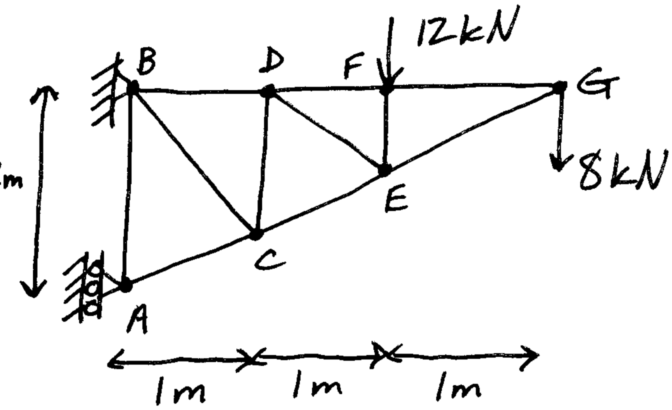
$$+ \frac{\sqrt{2}}{2}\left[200 + 1800\right] = -771.7\ \text{lb}\cdot\text{in}$$

$$\rightarrow \vec{M}_{CD} = {}^{-}771.7\left(\frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j}\right) \text{ lb}\cdot\text{in}$$

$$\boxed{\vec{M}_{CD} = -545.7\,\vec{i} - 545.7\,\vec{j} \text{ lb}\cdot\text{in.}}$$

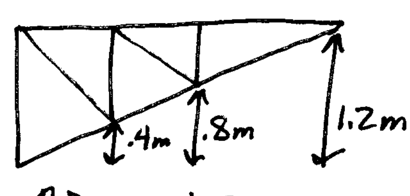
* A better choice of origin would have been directly under the center of the spool. This point is still on the axis CD, but the moment due to the weight W about this point is zero.

c)

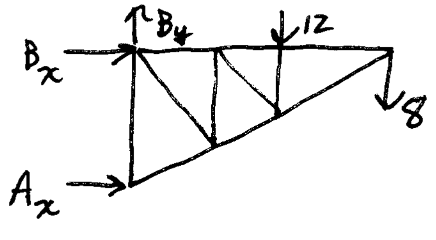


Forces carried by each member? Method of sections for DF, EC, DE.

Some geometry

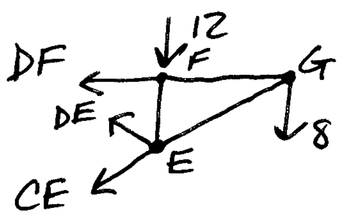


FBD of structure:



$$\Sigma F_x = 0 = A_x + B_x \quad \rightarrow \quad B_x = {}^{-}40 \text{ kN}$$
$$\Sigma F_y = B_y - 12 - 8 = 0 \rightarrow B_y = 20 \text{ kN}$$
$$\Sigma M_z^B = 1.2 A_x - 2\cdot 12 - 3\cdot 8 = 0 \rightarrow A_x = 40 \text{ kN}$$

Section through DF, EC, DE



[redacted] $\Sigma M_z^E = -8 + 0.4\,DF = 0$

$\rightarrow DF = 20\,kN$ (tension)

$\Sigma F_x = -DF - DE\frac{1}{\sqrt{1.16}} - CE\frac{1}{\sqrt{1.16}} = 0$

$$\Sigma F_y = -12 - 8 + DE\frac{.4}{\sqrt{1.16}} - CE\frac{.4}{\sqrt{1.16}} = 0$$

$$\therefore CE = DE - 20\frac{\sqrt{1.16}}{.4}$$

$$\rightarrow -20 - DE\frac{1}{\sqrt{1.16}} - DE\frac{1}{\sqrt{1.16}} + 20\frac{\sqrt{1.16}}{.4}\frac{1}{\sqrt{1.16}} = 0$$

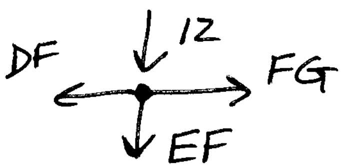
$$\therefore DE = 20\left(-1 + \frac{1}{.4}\right)\frac{\sqrt{1.16}}{2} = 16.16\,kN$$

(tension)

$$\rightarrow CE = -37.7\,kN \text{ (compression)}$$

Method of joints on the rest

F:

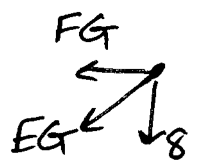


$\Sigma F_y = -12 - EF = 0 \rightarrow EF = -12\,(c)$

$\Sigma F_x = FG - DF = 0$

$\rightarrow FG = DF = 20\,(t)$

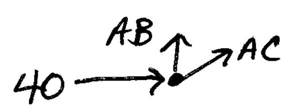
G:



$\Sigma F_x = -FG - EG\frac{1}{\sqrt{1.16}} = 0$

$\rightarrow EG = -20\sqrt{1.16} = -21.54\,(c)$

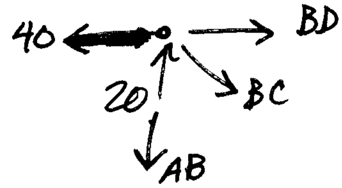
A:



$\Sigma F_x = 40 + AC\frac{1}{\sqrt{1.16}} = 0 \Rightarrow AC = -40\sqrt{1.16} = -43.08\,(c)$

$\Sigma F_y = AB + AC\frac{.4}{1.16} = 0 \Rightarrow AB = +40 \cdot .4 = +16\,(t)$

B:



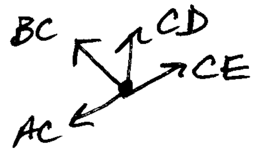
$$\Sigma F_x = BD - 40 + BC\frac{1}{\sqrt{1.64}} = 0$$

$$\Sigma F_y = -AB + 20 - BC\frac{.8}{\sqrt{1.64}} = 0$$

$$\rightarrow BC = 4\frac{\sqrt{1.64}}{.8} = 6.4\ (t)$$

$$\rightarrow BD = 40 - \frac{4}{0.8} = 35\ (t)$$

C:



$$\Sigma F_x = CE\frac{1}{\sqrt{1.16}} - AC\frac{1}{\sqrt{1.16}} - BC\frac{1}{\sqrt{1.64}} = 0$$

$$= -37.7\frac{1}{\sqrt{1.16}} + 40\sqrt{1.16}\frac{1}{\sqrt{1.16}} - \frac{4}{.8}\frac{\sqrt{1.64}}{\sqrt{1.64}} = 0$$

$$= -35 + 40 - 5 = 0 \checkmark$$

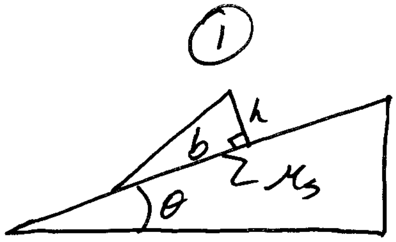
$$\Sigma F_y = CD + CE\frac{.4}{\sqrt{1.16}} - AC\frac{.4}{\sqrt{1.16}} + BC\frac{.8}{\sqrt{1.64}} = 0$$

$$\rightarrow CD = 35(.4) - 40(.4) - 4 = -6\ (c)$$

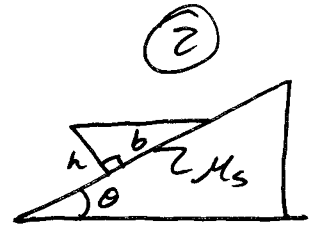
All values are in kN.

AB = 16 kN (tension)
AC = -43.1 kN (compression)
BC = 6.4 kN (tension)
BD = 35 kN (tension)
CD = -6 kN (compression)
CE = -37.7 kN (compression)
DE = 16.16 kN (tension)
DF = 20 kN (tension)
EF = -12 kN (compression)
EG = -21.5 kN (compression)
FG = 20 kN (tension)

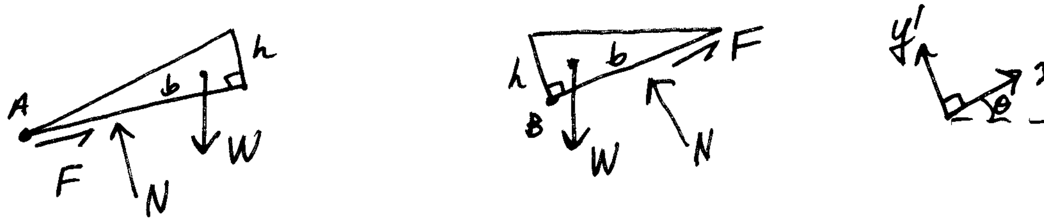
D)



versus



FBDs



In both cases W acts at the centroid of the triangle which is located at $\frac{b}{3}$ and $\frac{h}{3}$ from the corner with the right angle.

Analysis: $\Sigma F_{x'} = F - W\sin\theta = 0$
$\Sigma F_{y'} = N - W\cos\theta = 0$ } True for case 1 or 2

$\therefore \quad F = W\sin\theta \quad , \quad N = W\cos\theta$

Impending Sliding occurs if $F = \mu_s N$

$\therefore$ impending sliding when $\tan\theta = \mu_s$

$\tan\theta < \mu_s \rightarrow$ no sliding

At impending tipping both F and N act through the corner farthest down the inclined plane.

$\therefore$ Case 1 $\quad \sum M_{z'}^{A} = -\frac{2}{3} b\, W\cos\theta + \frac{h}{3} W\sin\theta = 0$

$\rightarrow \tan\theta = \frac{2b}{h}$ for impending tipping

Case 2 $\quad \sum M_{z'}^{B} = -\frac{1}{3} b\, W\cos\theta + \frac{h}{3} W\sin\theta = 0$

$\rightarrow \tan\theta = \frac{b}{h}$ for impending tipping

Case 1 : $\mu_s > \frac{2b}{h} \rightarrow$ tips first

$\mu_s < \frac{2b}{h} \rightarrow$ slides first

Case 2 : $\mu_s > \frac{b}{h} \rightarrow$ tips first

$\mu_s < \frac{b}{h} \rightarrow$ slides first