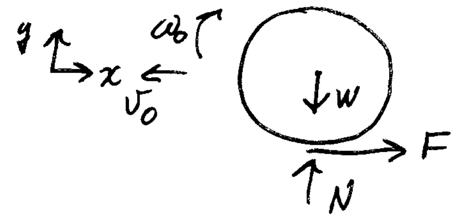
Fall 2003 Test 3 Solutions

1) FBD:



$\Sigma F_y = N - W = 0$

$\rightarrow N = W$

$F = \mu N = \mu W$

Linear impulse & momentum in x

$$\int_0^{\Delta t} F_x^{ext}\, dt = m v_x^f - m v_x^i$$

$$\mu W \Delta t = 0 - \frac{W}{g}(-v_0) \rightarrow \Delta t = \frac{v_0}{\mu g}$$

Angular impulse & angular momentum in z

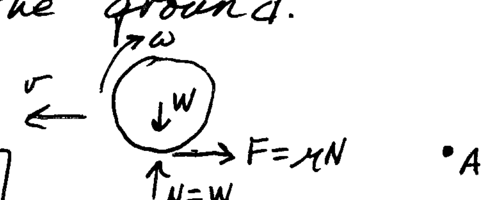
$$\int_0^{\Delta t} M_z^{cm}\, dt = h_z^{cmf} - h_z^{cmi}$$

$$\mu W R \Delta t = 0 - \frac{2}{5}\frac{W}{g}R^2(-\omega_0)$$

$$\cancel{\mu} W \cancel{R} \frac{v_0}{\cancel{\mu} \cancel{g}} = \frac{2}{5}\frac{\cancel{W}}{\cancel{g}} R^{\cancel{2}} \omega_0$$

$$\boxed{\omega_0 = \frac{5}{2}\frac{v_0}{R}}$$

Or pick any <u>fixed</u> point A on the ground.



$$\int_0^{\Delta t} M_z^A\, dt = h_z^{Af} - h_z^{Ai}$$

$$0 = 0 - \left[I_{cm}(-\omega_0) + \frac{W}{g} v_0 R\right]$$

$$0 = \frac{2}{5}\frac{\cancel{W}}{\cancel{g}} R^{\cancel{2}} \omega_0 - \frac{\cancel{W}}{\cancel{g}} v_0 \cancel{R}$$

$$\rightarrow \boxed{\omega_0 = \frac{5}{2}\frac{v_0}{R}}$$

This problem can be solved with a ~~[illegible]~~ third approach.

Recall from page 192 of the class notes:

$$\vec{M}_A = \frac{d}{dt}\left[\vec{r}_{cm/A} \times m\vec{v}_{cm} + I_{cm}\omega\vec{k}\right] + v_A \times m\vec{v}_{cm}$$

Take A to be the point in contact with the ground. Then $\vec{M}_A = 0$. Also,

$$\vec{v}_A = \underbrace{\vec{v}_{cm}}_{v_{cm}\vec{i}} + \underbrace{\omega\vec{k} \times \vec{r}_{A/cm}}_{\omega r\vec{i}}$$

$\rightarrow$ both $\vec{v}_A$ and $\vec{v}_{cm}$ are in the $\vec{i}$ direction and therefore $\vec{v}_A \times m\vec{v}_{cm} = 0$

$$\rightarrow \frac{d}{dt}\Big[\underbrace{\vec{r}_{cm/A} \times m\vec{v}_{cm} + I_{cm}\omega\vec{k}}_{h_z^A\vec{k}}\Big] = 0$$

$$\rightarrow h_z^{Af} = h_z^{Ai} \quad \text{with} \quad v_{cm}^f = 0,\ \omega_f = 0$$

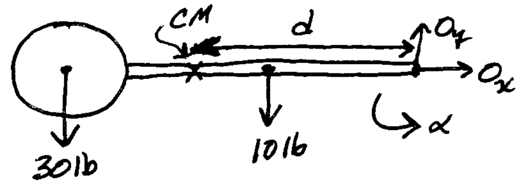
$$0 = r\vec{j} \times m(-v_0\vec{i}) + \frac{2}{5}mr^2(-\omega_0)\vec{k}$$

$$0 = \cancel{m}v_0\cancel{r}\vec{k} - \frac{2}{5}\cancel{m}\cancel{r}\omega_0 r\vec{k}$$

$$\therefore \boxed{\omega_0 = \frac{5}{2}\frac{v_0}{r}}$$

But angular momentum in z about point A is conserved because $\vec{v}_A$ is parallel to $\vec{v}_{cm}$, <u>not</u> because A is a fixed point. This needs to be realized in order to receive full credit.

2) FBD a) $\overset{y}{\uparrow}\!\!\rightarrow x$



$$\Sigma F_x = O_x = m a_{cmx}$$
$$\Sigma F_y = O_y - 40 = m a_{cmy}$$
$$\Sigma M_z^o = 30 \cdot 3 + 10 \cdot 1 = I_o \alpha$$

$$d = \frac{10 \cdot 1 + 30 \cdot 3}{40} = \frac{5}{2} \text{ ft.}$$

$$\vec{a}_{cm} = \underbrace{\cancel{\vec{a}_o}}_{O \text{ fixed}} + \vec{\alpha} \times \vec{r}_{cm/o} + \underbrace{\cancel{\vec{\omega}} \times (\vec{\omega} \times \vec{r}_{cm/o})}_{O \text{ starts from rest}}$$
$$a_{cmx}\vec{i} + a_{cmy}\vec{j} = -\alpha d \vec{j}$$

$$\rightarrow \quad a_{cmx} = 0 \quad , \quad a_{cmy} = -\alpha d$$

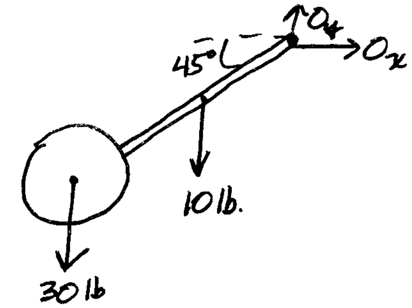
$$\rightarrow \boxed{O_x = 0} \qquad I_o = \frac{2}{5}\frac{30}{32.2}1^2 + \frac{30}{32.2}3^2 + \frac{1}{12}\frac{10}{32.2}2^2 + \frac{10}{32.2}1^2$$
$$= 9.172 \text{ slugs} \cdot \text{ft}^2$$

$$\rightarrow \alpha = \frac{100}{9.172} = 10.903 \text{ rad}/s^2$$
$$\rightarrow a_{cmy} = -27.257 \text{ ft}/s^2$$

$$\rightarrow O_y = 40 + \frac{40}{32.2}(-27.257)$$

$$\boxed{O_y = 6.14 \text{ lbs}}$$

FBD b) $\uparrow^{y}\rightarrow_{x}$



$$\Sigma F_x = O_x = m a_{cmx}$$
$$\Sigma F_y = O_y - 40 = m a_{cmy}$$
$$\Sigma M_z^O = 30 \cdot 3 \tfrac{\sqrt{2}}{2} + 10 \cdot 1 \tfrac{\sqrt{2}}{2} = I_o \alpha \rightarrow \alpha = 7.7095 \tfrac{rad}{s^2}$$

$$\vec{a}_{cm} = \overset{O \text{ fixed}}{\cancel{\vec{a}_o}} + \vec{\alpha} \times \vec{r}_{cm/o} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{cm/o})$$
$$a_{cmx}\vec{i} + a_{cmy}\vec{j} = \alpha \vec{k} \times d\left(-\tfrac{\sqrt{2}}{2}\vec{i} - \tfrac{\sqrt{2}}{2}\vec{j}\right) + \omega^2 d\left(\tfrac{\sqrt{2}}{2}\vec{i} + \tfrac{\sqrt{2}}{2}\vec{j}\right)$$
$$= \alpha d\left(\tfrac{\sqrt{2}}{2}\vec{i} - \tfrac{\sqrt{2}}{2}\vec{j}\right) + \omega^2 d\left(\tfrac{\sqrt{2}}{2}\vec{i} + \tfrac{\sqrt{2}}{2}\vec{j}\right)$$

$$\rightarrow a_{cmx} = \tfrac{\sqrt{2}}{2}(\alpha d + \omega^2 d), \quad a_{cmy} = \tfrac{\sqrt{2}}{2}(-\alpha d + \omega^2 d)$$

Conservation of energy: $\cancel{KE_i} + \cancel{PE_i} + \cancel{W_{NC}} = KE_f + PE_f$

$$0 = \tfrac{1}{2} I_o \omega^2 - \underbrace{mg}_{W} d \tfrac{\sqrt{2}}{2}$$

$$\omega = \sqrt{\frac{\sqrt{2}\, Wd}{I_o}} = \sqrt{\frac{\sqrt{2}\; 40 \cdot 5/2}{9.172}} = 3.927 \tfrac{rad}{s}$$

$$\rightarrow a_{cmx} = 40.89 \tfrac{ft}{s^2}, \quad a_{cmy} = 13.63\ ft/s^2$$

$$\rightarrow \boxed{O_x = 50.8 \text{ lb.} \qquad O_y = 56.9 \text{ lb.}}$$

$$\vec{v}_{center} = \overset{O \text{ fixed}}{\cancel{\vec{v}_o}} + \vec{\omega} \times \vec{r}_{center/o} = \omega \vec{k} \times 3\left(-\tfrac{\sqrt{2}}{2}\vec{i} - \tfrac{\sqrt{2}}{2}\vec{j}\right)$$

$$\boxed{\vec{v}_{center} = \omega\, 3\left(\tfrac{\sqrt{2}}{2}\vec{i} - \tfrac{\sqrt{2}}{2}\vec{j}\right) = 11.78\left(\tfrac{\sqrt{2}}{2}\vec{i} - \tfrac{\sqrt{2}}{2}\vec{j}\right)}$$

3) $\vec{a}_B = \cancel{\vec{a}_C}^{\,0} + \cancel{\vec{\alpha}_{BC}}^{\,0} \times \vec{r}_{B/C} + \vec{\omega}_{BC} \times (\vec{\omega}_{BC} \times \vec{r}_{B/C})$

$a_{Bx}\vec{i} + a_{By}\vec{j} = 4(0.06\vec{i} - 0.03\vec{j}) = 0.24\vec{i} - 0.12\vec{j}$

$\vec{v}_B = \cancel{\vec{v}_C}^{\,0} + \vec{\omega}_{BC} \times \vec{r}_{BC} = 2\vec{k} \times (-0.06\vec{i} + 0.03\vec{j})$

$v_{Bx}\vec{i} + v_{By}\vec{j} = -0.06\vec{i} - 0.12\vec{j}$

$\vec{v}_B = \cancel{\vec{v}_D}^{\,0} + \vec{v}_{B/rect} + \vec{\omega}_{rect} \times \vec{r}_{B/D}$

$-0.06\vec{i} - 0.12\vec{j} = v\vec{j} + \omega_{rect}\vec{k} \times (0.04\vec{i} + 0.03\vec{j})$

$= -0.03\,\omega_{rect}\vec{i} + (v + 0.04\omega_{rect})\vec{j}$

$\therefore \; \omega_{rect} = 2\,\frac{rad}{s}, \quad v = -0.2\,\frac{m}{s}$

$\vec{a}_B = \cancel{\vec{a}_D}^{\,0} + \vec{a}_{B/rect} + 2\vec{\omega}_{rect} \times \vec{v}_{B/rect} + \vec{\alpha}_{rect} \times \vec{r}_{B/D} + \vec{\omega}_{rect} \times (\vec{\omega}_{rect} \times \vec{r}_{B/D})$

$= a\vec{j} - \frac{v^2}{R}\vec{i} + 2 \cdot 2\vec{k} \times v\vec{j} + \alpha_{rect}\vec{k} \times (0.04\vec{i} + 0.03\vec{j})$

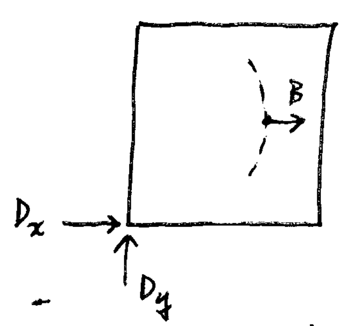
$+ 4(-0.04\vec{i} - 0.03\vec{j})$

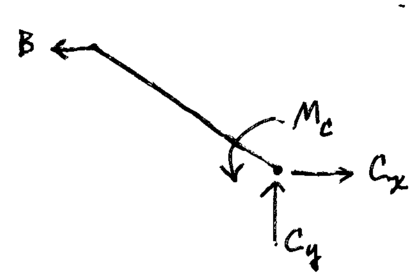
$\vec{i}: \; 0.24 = -\frac{0.04}{0.04} + 0.8 - 0.03\,\alpha_{rect} - 0.16$

$\therefore \; \alpha_{rect} = -20 \; rad/s^2$

$\vec{j}$: can be used to determine a, but this is not needed to get the forces & moments

FBDs:





$$\text{Rod: } \quad \Sigma F_x = C_x - B = m_{BC}\, a^{BC}_{cmx} = 0.5(0.12) \qquad \Sigma F_y = C_y = m_{BC}\, a^{BC}_{cmy} = 0.5(-0.06) \qquad \vec{a}^{BC}_{cm} = \frac{\vec{a}_B}{2}$$

$$\Sigma M^c_z = M_c + B(0.03) = I_c \cancelto{0}{\alpha_{BC}}$$

$$\text{Rect: } \quad \Sigma F_x = D_x + B = m_{rect}\, a^{rect}_{cmx} = 1(0.5) = 0.5$$

$$\Sigma F_y = D_y = m_{rect}\, a^{rect}_{cmy} = 1(-0.62) = -0.62$$

$$\Sigma M^D_z = -B(0.03) = I_D\, \alpha_{rect}$$

$$I_D = \frac{1}{12} m_{rect}(0.05^2 + 0.06^2) + m_{rect}(0.025^2 + 0.03^2)$$

$$= 0.002033 \text{ kg}\cdot\text{m}^2$$

$$\therefore \boxed{B = 1.356\text{ N} \rightarrow C_x = 1.416\text{ N} \qquad C_y = -0.03\text{ N} \qquad M_c = -0.04067\text{ N}\cdot\text{m}}$$

$$\vec{a}^{rect}_{cm} = \cancelto{0}{\vec{a}_D} + \vec{\alpha}_{rect} \times \vec{r}_{CM/D} + \vec{\omega}_{rect} \times (\vec{\omega}_{rect} \times \vec{r}_{CM/D})$$

$$= -20\vec{k} \times (0.025\vec{i} + 0.03\vec{j}) + \omega^2_{rect}(-0.025\vec{i} - 0.03\vec{j})$$

$$= -0.5\vec{j} + 0.6\vec{i} - 0.1\vec{i} - 0.12\vec{j}$$

$$\vec{a}^{rect}_{cm} = 0.5\vec{i} - 0.62\vec{j}$$

$$\boxed{D_y = -0.62\text{ N} \qquad D_x = -0.856\text{ N}}$$