Math 211
Second Midterm
April 1, 2003

Make sure to show your work and justify your arguments.

Calculator policy: You may NOT use a calculator on this exam. The reasons are twofold: first, the calculations are not hard and you will be able to do them in your head. Second, some calculators are equipped with programs which evaluate determinants and do matrix operations.

1) (15p) Find the solution of the linear system of equations

$$\begin{aligned} x + 2y - 3z + 5v + 3w &= 10 \\ 2x + 4y - 6z + 11v + 8w &= 23 \\ -x - 2y + 3z - 3v + w &= -4. \end{aligned}$$

Help: using MATLAB, the row reduced echelon form of the augmented matrix of this system is

$$\begin{bmatrix} 1 & 2 & -3 & 0 & -7 & -5 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2) (20p) Consider the system of linear equations

$$\begin{aligned} kx + y + z &= 1 \\ x + ky + z &= 1 \\ x + y + kz &= 1. \end{aligned}$$

(k is a real constant.) For which values of k does the system have (a) a unique solution, (b) no solution, (c) infinitely many solutions?

3) (20p) Find those x values for which the matrix

$$\begin{bmatrix} 2 & -1 & 3 \\ 5 & -1 & x \\ x & 1 & -1 \end{bmatrix}$$

is nonsingular.

4) Consider the system of ODE's

$$\begin{aligned} x' &= x - x^4 + y^2 \\ y' &= -2x^2 + 4y. \end{aligned}$$

a)(5p) Verify that $x = e^t$, $y = e^{2t}$ is a solution.
b)(5p) Verify that $x = -e^t$, $y = e^{2t}$ is a solution.
b)(5p) Consider the solution to the system with initial conditions $x(0) = 0$, $y(0) = 1$. Is there a time T such that $x(T) = 2$ and $y(T) = 1$? Explain.

EXAM CONTINUES ON BACK PAGE

5) (15p) Consider the three vectors

$$\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} \quad \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}.$$

Are these vectors linearly independent? If not, express the zero vector as a nontrivial linear combination of these vectors.

6) (15p) Check that $\det(A)\det B = \det(AB)$ for the matrices

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 1 & 2 \\ 0 & -1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 4 \\ 1 & 1 & 1 \end{bmatrix}.$$