

Math 211
Exam # 2

April 3, 2001

Instructions: This is a closed book, 75 minute exam. Write out and sign the honor pledge on your exam book. In addition please print your name on your exam book.

You are allowed to use a calculator to do simple computations. You are **not** allowed to use a calculator for any symbolic computations such as computing derivatives or integrals, or to solve differential equations.

Please give reasons for all of your answers.

1. Consider the system of differential equations

$$\begin{aligned}x_1' &= x_1 + x_2 \\x_2' &= -x_1 + t^2.\end{aligned}$$

- a) (5 points) Is the pair of functions $x_1(t) = t^2$, $x_2(t) = t - 3$ a solution?
b) (5 points) Is the pair of functions $x_1(t) = t^2 + 2t$, $x_2(t) = -t^2 + 2$ a solution?

2. (10 points) Is the matrix

$$A = \begin{pmatrix} 3 & -2 \\ -1 & 0 \end{pmatrix}$$

singular or nonsingular? Is invertible? Does it have a nontrivial nullspace?

3. Consider the system $\mathbf{y}' = A\mathbf{y}$, where

$$A = \begin{pmatrix} -2 & 1 \\ -1 & 0 \end{pmatrix}.$$

- a) (5 points) Show that

$$\mathbf{y}_1(t) = e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \text{and} \quad \mathbf{y}_2(t) = e^{-t} \begin{pmatrix} 1-t \\ -t \end{pmatrix}$$

are both solutions.

- b) (10 points) Assuming they are solutions, show that \mathbf{y}_1 and \mathbf{y}_2 form a fundamental set of solutions.
c) (10 points) Assuming the answer to part b), find the solution to the initial value problem $\mathbf{y}' = A\mathbf{y}$, with $\mathbf{y}(0) = (1, 5)^T$.

(More on the other side)

4. Consider the system

$$\begin{aligned}x' &= -y + x(4 - x^2 - y^2) \\y' &= x + y(4 - x^2 - y^2)\end{aligned}$$

- (5 points) Show that $x(t) = 2 \cos t$, $y(t) = 2 \sin t$ is a solution.
- (5 points) Sketch the solution found in part a) in the phase plane.
- (5 points) Suppose we have another solution $x(t)$, $y(t)$ which satisfies the initial conditions $x(0) = 1$ and $y(0) = 1$. Show that $x^2(t) + y^2(t) < 4$ for all t .

5. (10 points) Describe the type (i.e., saddle point, nodal source, etc.) of the equilibrium point at $\mathbf{0}$ for the system $\mathbf{x}' = A\mathbf{x}$ in each of the following cases

a) $A = \begin{pmatrix} 5 & 6 \\ -4 & -5 \end{pmatrix}$

b) $A = \begin{pmatrix} -1 & -4 \\ 2 & 5 \end{pmatrix}$

c) $A = \begin{pmatrix} -5 & -3 \\ 6 & 1 \end{pmatrix}$

d) $A = \begin{pmatrix} -5 & -5 \\ 10 & 5 \end{pmatrix}$

6. Consider the system $\mathbf{y}' = A\mathbf{y}$, where

$$A = \begin{pmatrix} -2 & 0 & 0 \\ 2 & -3 & -2 \\ -2 & 4 & 3 \end{pmatrix}.$$

- (10 points) Describe the main steps in procedure that you follow to find a fundamental set of solutions for a system like this.
- (20 points) Find a fundamental set of solutions. Be sure to point out why they form a fundamental set of solutions.