

Math 211 Exam 2

November 13, 2003

Instructions.

- You have 75 minutes to complete the exam, so budget your time so that you will be able to attempt all problems.
- Print your name, the instructor's name and the section number on the blue book.
- Show all your work. Answers without proper work will not receive full credit.
- No calculators are allowed
- Upon finishing please write and sign your pledge on front page of your blue book: On my honor I have neither given nor received any aid on this exam.

1. (20 points) Let

$$A = \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix}.$$

- (a) What is the dimension of $\text{null}(A)$?
- (b) Find a basis for $\text{null}(A)$.
- (c) Suppose \mathbf{x}_p is a particular solution to $A\mathbf{x} = \mathbf{b}$ for some $\mathbf{b} \in \mathbf{R}^4$, find the general solution to $A\mathbf{x} = \mathbf{b}$.

2. (20 points)

- (a) Find those x values for which the following vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ x \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} x \\ 1 \\ 1 \end{bmatrix}$$

are linearly dependent.

- (b) For each x found in part (a), express the zero vector as a nontrivial linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

3. (20 points) Consider the system

$$\begin{aligned} x' &= 1 - (y - \sin x) \cos x \\ y' &= \cos x - y + \sin x. \end{aligned}$$

- (a) Show that $x(t) = t, y(t) = \sin t$ is a solution.
- (b) Plot the solution found in part (a) in the phase plane.
- (c) Consider the solution to the system with the initial conditions $x(0) = \pi/2$ and $y(0) = 0$. Show that $y(t) < \sin x(t)$ for all t .

4. (20 points)

- (a) Find the general solution to

$$\mathbf{y}' = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} \mathbf{y}.$$

- (b) Find all equilibrium points and plot them in the phase plane and sketch the solution curves. Use arrows to indicate direction of motion for all solutions.
- (c) Classify the equilibrium points as one of the 6 types discussed in class.
- (d) Classify the equilibrium points as unstable, stable but not asymptotically stable, or asymptotically stable.

5. (20 points) Consider the 3×3 matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}.$$

- (a) Find a fundamental set of solutions to the system $\mathbf{x}' = A\mathbf{x}$.
- (b) Solve the initial value problem $\mathbf{x}' = A\mathbf{x}$, $\mathbf{x}(0) = (1, 1, 1)^T$.