Math 211, Exam 2
November 14th, 2002

Instructions.

- You have 75 minutes to complete the exam, so budget your time so that you will be able to attempt all sections.
- Print your name and section on the EXAM BOOKLET.
- **Show all your work!** Answers without proper work will not receive full credit.
- No calculators are allowed.
- Put a box around your final answer.
- Submit this exam with your booklet when you are done.

Upon finishing PLEASE write and sign your pledge on front page of your exam booklet:
On my honor I have neither given nor received any aid on this exam.

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1. (a) **(15 points)** Find a parametric representation for the solution set of the system

$$\begin{aligned} x_1 + 2x_2 - 2x_3 + x_4 &= 2 \\ x_2 - 3x_3 - x_4 &= 3 \\ x_3 - x_4 &= 0 \end{aligned}$$

   (b) How would you (geometrically) describe the solution set? (Examples of geometrical description include a circle in $\mathbb{R}^2$, a line in $\mathbb{R}^3$, a hyperbola in $\mathbb{R}^2$, a plane in $\mathbb{R}^4$ and an empty set.)

2. **(15 points)** Which of the following matrices
   (a) is(are) singular?
   (b) have non-trivial nullspace?
   (c) have determinant $= 0$?

Justify your answers.

$$A = \begin{pmatrix} 1 & 3 & -2 \\ 2 & 5 & -3 \\ -3 & 2 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & -6 \\ -12 & 24 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 & 2 & 2 & 0 & 1 & 2 \\ 1 & 2 & 0 & 1 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 & 2 & 0 & 0 \\ 1 & 1 & 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

3. **(10 points)** Write the third order differential equation

$$y''' + y'y'' = \sin \omega t$$

with initial values

$$y(0) = 1, \; y'(0) = -1, \; y''(0) = 0$$

as a system of first order differential equations (DO NOT solve the system).

4. **(20 points)** Consider the system

$$\begin{aligned} x' &= y - x(x^2 + y^2 - 1) \\ y' &= -x - y(x^2 + y^2 - 1) \end{aligned}.$$

(a) Show that $x(t) = \sin t, \; y(t) = \cos t$ is a solution.
(b) Sketch the solution in part(a) in the phase plane.
(c) Consider the solution to the system with the initial conditions $x(0) = 0.5$ and $y(0) = 0$. Show that $x^2(t) + y^2(t) < 1$ for all t.

5. **(20 points)**
(a) Find the general solution to $\mathbf{y}' = A\mathbf{y}$ for

$$A = \begin{pmatrix} 2 & -6 \\ 0 & -1 \end{pmatrix}.$$

(b) Find all equilibrium points and plot them in the phase plane. Plot the half-line solutions (exponential solutions) and sketch enough solution curves to show what happens in every part of the phase plane. Use arrows to indicate direction of motion for all solutions.
(c) Classify the equilibrium points as one of the 6 types discussed in class.

6. **(20 points)** Consider the 3×3 matrix

$$A = \begin{pmatrix} -4 & 8 & 8 \\ -4 & 4 & 2 \\ 0 & 0 & 2 \end{pmatrix}.$$

(a) A has a complex eigenvalue $\lambda = 4i$, with its associated eigenvector $\mathbf{v} = (1 - i, 1, 0)^T$. Find **all** eigenvalues and associated eigenvectors.

(b) Find a fundamental set of real solutions to the system $\mathbf{x}' = A\mathbf{x}$.

(c) Solve the initial value problem

$$\mathbf{x}' = A\mathbf{x}, \quad \mathbf{x}(0) = (4, -1, 2)^T.$$