Math 211
Second Midterm
October 30, 2001

Make sure to show your work and justify your arguments.

Calculator policy: You may NOT use a calculator on this exam. The reasons are twofold: first, the calculations are not hard and you will be able to do them in your head. Second, some calculators are equipped with programs which evaluate determinants and do matrix operations.

1)(14p) Find the nullspace of the matrix

$$A = \begin{bmatrix} 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 4 & 0 & 2 \end{bmatrix}.$$

2) Consider the vectors $u = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ and $v = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$.

a)(7p) Write $w = \begin{bmatrix} 8 \\ -1 \\ 5 \end{bmatrix}$ as a linear combination of u and v.

b)(7p) Find k so that $w = \begin{bmatrix} 1 \\ -5 \\ k \end{bmatrix}$ is a linear combination of u and v.

3)(14p) Is the matrix

$$A = \begin{bmatrix} 0 & -2 & 4 \\ 1 & 0 & -1 \\ 0 & 4 & -7 \end{bmatrix}$$

invertible? If so, find A^{-1}.

4)(14p) Find the determinant of the matrix

$$A = \begin{bmatrix} -1 & 3 & 2 \\ 4 & -8 & 1 \\ 2 & 2 & 5 \end{bmatrix}.$$

5)(14p) Consider the following system in unknowns x and y:

$$\begin{aligned} x - ay &= 1 \\ ax - 4y &= b \end{aligned}$$

For which values of a and b does the system have a unique solution, no solution or infinitely many solutions?

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6) Consider the system of ODE's

$$\begin{aligned} x' &= -2y + x(x^2 + 4y^2 - 4) \\ y' &= \tfrac{1}{2}x - y(x^2 + 4y^2 - 4). \end{aligned}$$

a)(6p) Verify that $x = 2\cos t$, $y = \sin t$ is a solution. (See Figure 1. for the phase plane picture of this solution.)

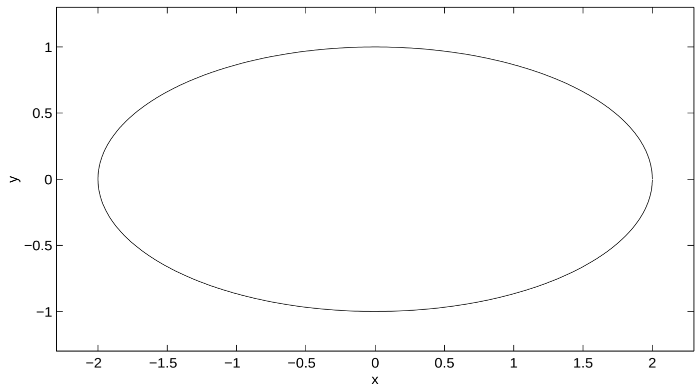

FIGURE 1. $x = 2\cos t, y = \sin t$

b)(8p) Consider the solution to the system with initial conditions $x(0) = 1$, $y(0) = 0$. Is there a time T such that $x(T) = 3$? Explain.

7) Consider the initial value problem

$$y' = t^2 \quad y(0) = 0$$

on the interval $[0, 1]$.

a)(5p) Find an exact solution $y = f(t)$ to this problem. Graph this solution over the interval $[0, 1]$.

Suppose we apply Euler's method to get a numerical solution:

t	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
y	$y_0 = 0$	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}

b)(5p) Compute y_2.

c)(6p) Will $y_k > f(t_k) = f(k/10)$ for any $k = 0, \ldots, 10$? Explain.