

Math 211

Exam # 1

February 6, 2001

Instructions: This is a closed book, 75 minute exam. Write out and sign the honor pledge on your exam book. In addition please print your name on your exam book.

You are allowed to use a calculator to do simple computations. You are **not** allowed to use a calculator for any symbolic computations such as computing derivatives or integrals, or to solve differential equations.

Please give reasons for all of your answers.

1. Consider the differential equation $x' = \frac{t}{x - t^2x}$.
 - a) (8 points) Find the general solution.
 - b) (8 points) Find the solution which satisfies $x(0) = 4$.
 - c) (4 points) What is the interval of existence for the solution you found in part b)?

2. Consider the differential equation $y' = \frac{2ty}{1 + t^2} + 4t$.
 - a) (8 points) Find the general solution.
 - b) (8 points) Find the solution which satisfies $y(0) = 13$.
 - c) (4 points) What is the interval of existence for the solution you found in part b)?

3. Consider a tank which originally contains 100 gallons of pure water. At time $t = 0$ two valves are opened. The first starts a flow of sugar water into the tank at a rate of 1 gallon per minute with a concentration of p lb/gal. The second starts a flow out of the tank at a rate of 2 gallons per minute. As usual we assume perfect and instantaneous mixing.
 - a) (6 points) What is the differential equation model for this situation? Include the proper initial condition.
 - b) (6 points) Find the general solution for the equation you found in part a).
 - c) (6 points) Find the particular solution that solves the initial value problem you found in part a).
 - d) (4 points) What is p if the concentration of the sugar solution is 1.8 lb/gal when there is 10 gallons left in the tank?

4. Consider a system of two large tanks, tank A and tank B connected by a pipe. Suppose that at time $t = 0$ tank A contains 200 gallons and tank B contains 300 gallons of salt solution at different concentrations. Suppose there is:

- a flow of a salt solution with concentration 2lb/gal into tank A at the rate of 10gal/min
- a flow of the salt solution from tank A to tank B at 10gal/min
- a flow out of tank B at the rate of 5gal/min.

Let $x(t)$ be the amount of salt in tank A and let $y(t)$ be the amount of salt in tank B at time t .

- (8 points) Set up a differential equation that models the rate of change of x .
- (8 points) Set up a differential equation that models the rate of change of y .

You are not required to solve the equations.

5. (12 points) Suppose that y is a solution to the initial value problem

$$y' = \frac{\cos t}{4 - \sin^2 t}(4 - y^2), \quad \text{with } y(0) = 1.$$

Show that

$$\sin t < y(t) < 2, \quad \text{for all } t.$$

Give all details of your argument.

6. Consider the differential equation

$$y' = 2ty^2 - 4t^4 - 2t^5.$$

- (5 points) Is the function $y_1(t) = 1 + t^2$ a solution?
- (5 points) Is the function $y_2(t) = t$ a solution?