Math 211
First Midterm
February 18, 2003

Make sure to show your work and justify your arguments.

Calculator policy: You may use calculators to evaluate standard functions on floating point numbers (like $\sqrt{3.12}$, $\ln(35/7)$, or $\sin(\pi/17)$). You may not use symbolic operations, numerical integration, or any graphing functions.

1) Suppose that $y(t) = 2e^{-4t}$ is the solution of the initial value problem

$$y' + ky = 0, \qquad y(0) = y_0.$$

What are the constants k and y_0? (14%)

2) Consider the differential equation

$$y' = \frac{1+y}{t^3}.$$

a) Find the general solution. (9%)

b) Find a particular solution with $y(1) = 0$ and identify its interval of existence. (6%)

3) Find a solution to the initial value problem

$$2y' + (\cos t)y = -3\cos t, \qquad y(0) = -4. \quad (14\%)$$

4) A tank originally contains 100 gal of fresh water. At time $t = 0$, a solution containing 0.2 lb of salt per gallon begins to flow into the tank at a rate of 3 gal/min and the well-stirred mixture flows out of the tank at the same rate.

(a) How much salt is in the tank after 10 min? (6%)

(b) Does the amount of salt approach a limiting value as time increases? If so, what is this limiting value? (6%)

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5) A model for population growth is given by the equation

$$P' = rP(\frac{P}{\theta} - 1)(1 - \frac{P}{K}),$$

where r, θ and K are given positive constants and $2\theta < K$.

a) Sketch the graph of the function on the right hand side of this differential equation and identify the equilibrium points. (7%)

b) Draw the phase line and analyze the stability near each equilibrium point. (7%)

c) Consider the solution $P(t)$ with initial value $P(0) = K/2$. Describe its behavior as $t \to \infty$. Does it approach any of the equilibrium solutions? (7%)

6) Can you conclude anything about the existence and uniqueness of the solution(s) of the initial value problem

$$y' = \frac{1+y}{t^3}, \qquad y(1) = 0? \quad (10\%)$$

7) Consider the initial value problem

$$y' = \frac{y-2}{\sin t - 2} \cos t, \qquad y(0) = 1.$$

Show that the solution $y(t)$ of this initial value problem satisfies

$$\sin t < y(t) < 2 \text{ for every } t. \quad (14\%)$$