Math 211, Exam 1
September 30th, 2003

Instructions.

- You have 75 minutes to complete the exam. Budget your time so that you will be able to attempt all sections.
- Print your name and section on the EXAM BOOKLET.
- **Show all your work!** Answers without proper work will not receive full credit.
- No calculators are allowed.
- Put a box around your final answer.

Upon finishing PLEASE write and sign your pledge on front page of your exam booklet:

On my honor I have neither given nor received any aid on this exam.

---

1. (14 points) Find the exact solution of the initial value problem, and indicate the interval of existence.
$$y' = \frac{(y^2+1)}{y}, \quad y(1) = 2.$$

2. (15 points)
(a) Find the general solution to the differential equation
$$ty' + y = 4t^2.$$
(b) Find the particular solution for the differential equation with initial value $y(1) = 3$. State the interval of existence of the solution.

3. (14 points) Consider the differential equation $y' = -2t\sqrt{1-y^2}$, where $\sqrt{\ }$ means the positive square root.
(a) Is $y(t) = \sin(t^2)$ a solution?
(b) Is $y(t) = \cos(t^2)$ a solution?

4. (14 points) Is it possible to find a function $f(t, x)$ that is continuous and has continuous partial derivatives such that the functions $x_1(t) = t$ and $x_2(t) = \sin t$ are both solutions to $x' = f(t, x)$ near $t = 0$? Why or why not? Explain your reasoning.

5. (14 points) A tank contains 100 gal of pure water. A salt solution with concentration 3 lb/gal enters the tank at a rate of 2 gal/min. Solution drains from the tank at a rate of 2 gal/min. Without solving the equation, use qualitative analysis to find the eventual concentration of the salt solution in the tank.

6. (14 points) For the initial value problem
$$y' = ty, \quad y(0) = 1,$$
use Euler's method to compute the first four iterations using step size $h = 1/3$. (i.e., calculate $y_0,\ y_1,\ y_2$, and y_3.)

7. (15 points) Consider the autonomous equation
$$y' = y(y-1)(y+2).$$
(a) Find and classify all equilibrium points.
(b) Draw the phase line.
(c) Sketch equilibrium solutions on $ty-$plane. Sketch at least one solution trajectory in each of the regions on $ty-$plane divided by the equilibrium solutions.