

Math 211, Exam 1

The use of calculators to do arithmetic, or to evaluate functions such as e^x , is permitted. However, the use of calculators for any other purpose, such as graphing or integrating, is banned.

If you use a fact which depends on a theorem, then you must say which theorem, and you must verify that the hypotheses of that theorem are satisfied.

Please make sure that your name, your professor's name, and your section number are all indicated clearly on your paper.

1. (15 points) Find the exact solution for the following initial value problem.

$$y' = \frac{(-2t)(1 + y^2)}{y}, \quad \text{with } y(0) = 1.$$

What is the interval of existence?

2. (10 points) Consider the differential equation $yy' = y^2 + e^{4t}$.
- Is $y(t) = e^{2t}$ a solution?
 - Is $y(t) = 2e^{2t}$ a solution?

3. (15 points) Consider the differential equation $x' - \frac{1}{t}x = t^4$.
- Find the general solution.
 - Find a particular solution satisfying $x(2) = 10$.

4. (10 points) Consider the initial value problem

$$y' = y^2, \quad y(0) = 1.$$

What is the approximate value of $y(1)$ given by Euler's method, with step size 0.5?

5. (20 points) Consider the autonomous equation $y' = (y + 1)(y^2 - 4)$.
- Find and classify the equilibrium points.
 - Draw the phase line.
 - On the same set of axes, sketch the equilibrium solutions, and
 - a solution satisfying $y(0) = 3$
 - a solution satisfying $y(0) = 0$
 - a solution satisfying $y(1) = 0$
 - a solution satisfying $y(0) = -3$

6. (15 points) Consider the differential equation $y' = y^2 - y - ye^{-t}$.
- a) Show that $y_1(t) = e^{-t}$ is a solution.
 - b) Find a solution of the form $y = \text{constant}$.
 - c) Sketch the solutions from part a) and b) on the same set of axes.
 - d) Suppose that $y(t)$ is a solution for which $0 \leq y(0) \leq 1$.
Show that $y(t) \rightarrow 0$ as $t \rightarrow \infty$.

7. (15 points) Suppose that you're currently paying \$1200/month to rent your 2-bedroom house. If you were to put that rent money toward a house mortgage instead, what price of house could you afford? Assume a 30 year mortgage with a 7% interest rate, compounded continuously, and that your payments are made continuously, at a constant rate.

Note: A "mortgage" just means a loan from the bank. You must pay interest on the loan at a rate of 7%, and the loan must be completely paid off at the end of 30 years.