Math 211

Exam # 2

April 11, 2000

Instructions: This is a closed book, 75 minute exam. Write out and sign the honor pledge on your exam book. In addition please print your name on your exam book.

You are allowed to use a calculator to do simple computations. You are **not** allowed to use a calculator for any symbolic computations such as computing derivatives or integrals, or to solve differential equations.

Please give reasons for all of your answers.

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1. Let

$$A = \begin{pmatrix} 0 & 3 & 4 & 2 \\ 1 & 0 & 5 & 3 \\ 1 & 0 & 1 & 2 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 4 \\ 16 \\ 11 \end{pmatrix}.$$

a) (10 points) Find the null space of A.

b) (10 points) Find the solution space for the system $A\mathbf{x} = \mathbf{b}$.

2. Consider the vectors

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 0 \\ 8 \\ 6 \end{pmatrix} \quad \text{and} \quad \mathbf{v}_3 = \begin{pmatrix} 4 \\ -24 \\ -34 \end{pmatrix}.$$

a) (10 points) Are the vectors $\mathbf{v}_1,\ \mathbf{v}_2$ and $\mathbf{v}_3$ linearly independent?

b) (5 points) Find a basis for the span of the vectors $\mathbf{v}_1,\ \mathbf{v}_2,$ and $\mathbf{v}_3$.

c) (10 points) Let

$$\mathbf{v}_4 = \begin{pmatrix} 4 \\ -24 \\ -35 \end{pmatrix}.$$

Are the vectors $\mathbf{v}_1,\ \mathbf{v}_2$ and $\mathbf{v}_4$ linearly independent?

(More on the other side)

3. Consider the system of differential equations $\mathbf{y}' = A\mathbf{y}$, where

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

a) (5 points) Show that

$$\mathbf{y}_1(t) = \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} \quad \text{and} \quad \mathbf{y}_2(t) = \begin{pmatrix} \sin t \\ \cos t \end{pmatrix}$$

are both solutions to the system.

b) (8 points) Show that $\mathbf{y}_1$ and $\mathbf{y}_2$ form a fundamental set of solutions.

c) (5 points) What is the general solution to the system?

d) (7 points) Find the particular solution $\mathbf{y}(t)$ to the system which satisfies the initial condition

$$\mathbf{y}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

4. Let

$$A = \begin{pmatrix} -3 & -1 \\ 2 & 0 \end{pmatrix}.$$

a) (5 points) Show that A has eigenvalues -1 and -2.

b) (10 points) For each of these eigenvalues find an eigenvector.

c) (10 points) Find a fundamental set of solutions for the system $\mathbf{y}' = A\mathbf{y}$.

5. (5 points) Compute the determinant of

$$A = \begin{pmatrix} 1 & 2 & 0 & 4 \\ -1 & 0 & 1 & 0 \\ 0 & 3 & 4 & 5 \\ -3 & 0 & 0 & -2 \end{pmatrix}.$$