Math 211

Exam # 2

November 2, 1998

Part 2

Instructions: Part 2 of Exam #2 is an open book, open notes, untimed, take home exam. It is due in the Mathematics Department Office by 4:00 PM on Friday, November 6. You may not discuss the exam with your fellow students. If you have any questions, consult one of the instructors for the course.

Write out and sign the pledge on your submitted solutions. Be sure to put your name on your submission. In addition put the name of your instructor at the top of the first page of your submission.

Please give reasons for all of your answers. Remember that some reasons are better than others. For example, it is better to refer to a theorem stated in class and/or in the books than to say "the computer printout shows that ...". Of course sometimes the computer printout is all you have.

In answering these questions you are not limited to the use of MATLAB, although it will be useful. You should use any combination of the analytic, qualitative, or numerical methods you have learned.

Your answers should consist of complete sentences organized into coherent paragraphs. This does not mean that they have to be long. Brevity is frequently a sign of understanding. For much of this exam you can record what you are doing in a MATLAB diary file, and use that as a start of your exam submission. MATLAB graphics should be numbered and referred to by that number. Any graphic which is not referred to will not be counted as part of your submission.

This exam contains large matrices and vectors. To eliminate mistakes that are inevitable when copying these to your computer we have automated the process. If you are working on an owlnet computer, you need only execute `exam2` at the MATLAB prompt and all of the vectors and matrices will entered for you. If you are working outside of owlnet, go to the Math 211 homepage and to the description of the second exam. There is a link there to **exam2.m**. Click on the link and you will see the file. Select and copy all of the commands, and paste them to the MATLAB Command Window. Hit Enter, and all of the data will be in the MATLAB workspace. Execute the MATLAB command `who` to be sure.

Notice that the points on the questions add to 40.

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1. Consider the matrix

$$A = \begin{pmatrix} 9 & 9 & -8 & -7 & -4 & -1 \\ 5 & 9 & -2 & -5 & -6 & 9 \\ -9 & -9 & 8 & 7 & -9 & 0 \\ -18 & -18 & 16 & 14 & -31 & -1 \end{pmatrix}.$$

a) (6 points) Describe the nullspace of A.

b) (8 points) Describe the solution space of $A\mathbf{x} = \mathbf{b}_1$, and that of $A\mathbf{x} = \mathbf{b}_2$, where

$$\mathbf{b}_1 = \begin{pmatrix} -33 \\ -17 \\ 0 \\ -23 \end{pmatrix} \quad \text{and} \quad \mathbf{b}_2 = \begin{pmatrix} -4 \\ 4 \\ -36 \\ -112 \end{pmatrix}.$$

2. (6 points) Consider the following vectors

$$\mathbf{v}_1 = \begin{pmatrix} 10 \\ -5 \\ 3 \\ 0 \\ 8 \\ 6 \\ 0 \end{pmatrix} \quad \mathbf{v}_2 = \begin{pmatrix} -9 \\ 7 \\ -1 \\ 3 \\ 6 \\ 9 \\ 5 \end{pmatrix} \quad \text{and} \quad \mathbf{v}_3 = \begin{pmatrix} -6 \\ -1 \\ 9 \\ 9 \\ -1 \\ 8 \\ -8 \end{pmatrix}.$$

Either show that they are linearly independent or find a linear combination which is equal to the zero vector.

3. Consider the vectors

$$\mathbf{u} = \begin{pmatrix} 2 \\ -7 \\ 9 \\ 7 \\ 5 \\ 6 \\ -3 \end{pmatrix} \quad \text{and} \quad \mathbf{w} = \begin{pmatrix} 54 \\ -28 \\ 2 \\ -15 \\ 13 \\ -8 \\ -2 \end{pmatrix}.$$

a) (4 points) Find a linear combination of the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ in Problem #2 which is equal to $\mathbf{u}$ or show that no such linear combination exists.

b) (4 points) Find a linear combination of the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ in Problem #2 which is equal to $\mathbf{w}$ or show that no such linear combination exists.

4. Imagine a spring (with spring constant k_1) attached to a hook in the ceiling with an object of mass m_1 attached to the spring. Now imagine a second spring (with spring constant k_2) attached to the bottom of mass m_1, with a second object with mass m_2, attached to the second spring. This is an example of a *coupled oscillator*. If x and y represent the displacements of masses m_1 and m_2 from their respective equilibrium positions, then the second order system

$$\begin{aligned} m_1 x'' &= -k_1 x + k_2(y - x), \\ m_2 y'' &= -k_2(y - x), \end{aligned}$$

models the coupled oscillator.

a) (6 points) Set $x_1 = x$, $x_2 = x'$, $x_3 = y$, and $x_4 = y'$. Show that first order system

$$\begin{aligned}
{x_1}' &= x_2, \\
{x_2}' &= -\frac{k_1}{m_1} x_1 + \frac{k_2}{m_1}(x_3 - x_1), \\
{x_3}' &= x_4, \\
{x_4}' &= -\frac{k_2}{m_2}(x_3 - x_1).
\end{aligned}$$

is equivalent to the second order system for x and y in the sense that a solution of either system leads easily to a solution of the other system.

b) (6 points) Assume $k_1 = k_2 = 2$ and $m_1 = m_2 = 1$. Suppose that the first mass is displaced upward two units from its equilibrium position, and the second is dispaced downward two units from its. Suppose also that initially both masses are released from rest. Use `ode45` to solve for the motion and plot the position of each mass versus time on the same graph. Plot both curves on one figure, and properly label them.