Math 211

Exam # 1

February 29, 2000

Instructions: This is a closed book, 75 minute exam. Write out and sign the honor pledge on your exam book. In addition please print your name on your exam book.

You are allowed to use a calculator to do simple computations. You are **not** allowed to use a calculator for any symbolic computations such as computing derivatives or integrals, or to solve differential equations.

Please give reasons for all of your answers.

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1. (15 points) Consider the differential equation $x' = 3t^2(1 + x^2)$.

a) Find the general solution.

b) Find the solution which satisfies $x(0) = 0$.

c) What is the interval of existence for the solution you found in part b)?

2. (15 points) Consider the differential equation $y' = \cos(t) - y\tan(t)$.

a) Find the general solution.

b) Find the solution which satisfies $y(0) = 1$.

c) What is the interval of existence for the solution you found in part b)?

3. (18 points) Suppose $P(t)$ is the fish population (in millions) in Lake Houston at time t. We model this population with the differential equation

$$\frac{dP}{dt} = P(10 - P) - H$$

where H is a constant depending on the amount of fishing occurring in our region.

a) Draw the phase line for the case that no fishing occurs (i.e. $H = 0$). Sketch a few solutions, including the equilibrium solutions.

b) Draw the phase line for the case $H = 16$. Sketch a few solutions, including the equilibrium solutions.

c) Suppose the fish population has declined to $P(t_0) = 1$ at some time t_0. What values of H will ensure that the fish population won't go to zero?

4. (10 points) Consider the following system, consisting of two ponds, A and B, each of which contains a certain number of parasites.

- The volume of pond A is 1000 liters, and the volume of pond B is 500 liters.
- Parasites breed at a rate proportional to the number of parasites in the same pond.
- Clean water is flowing into pond A at 20 liters per minute.
- Well-mixed water flows from pond A to pond B at 20 liters per minute.
- Well-mixed water flows out of pond B at 20 liters per minute.

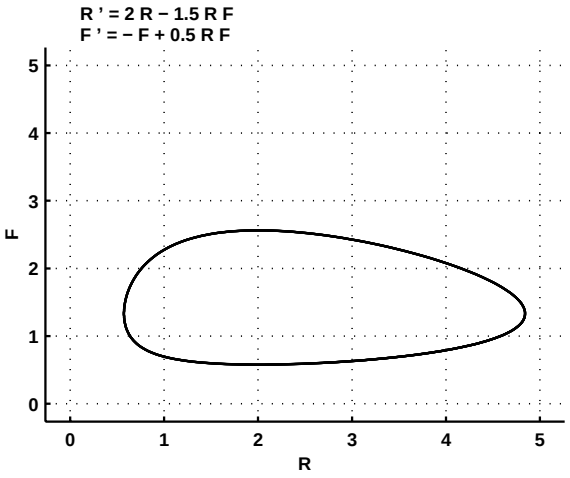
Set up a system of two differential equations modeling the numbers of parasites in ponds A and B at any time t. **You are not required to solve the system of differential equations.**

5. (10 points) Consider the following "predator-prey" model, where $R(t)$ denotes the population of rabbits and $F(t)$ denotes the population of foxes.

$$\frac{dR}{dt} = 2R - 1.5RF$$

$$\frac{dF}{dt} = -F + 0.5RF$$

A certain periodic solution to this system is plotted in the figure below.



The predator-prey model in Problem #5

Consider another solution, such that at some time t_0, the populations are given by $R(t_0) = 2$ and $F(t_0) = 2$. Will the population of rabbits ever exceed 5? Will the rabbits die out? Justify your answers.

6. (16 points) A credit card, issued by Duff beer company of Springfield to Homer Simpson charges 25% annually in interest on debt, compounded continuously. Suppose Homer has accumulated a debt of $6000 on his credit card. After getting a salary increase from Mr. Burns, Homer decides to repay his debt by making payments at the rate of $200 per month to the credit card company. He also decides not to make any more purchases.

a) Assuming that Homer's payments are made continuously, how long it will take him to repay his debt?

b) Over the period that it takes to repay his debt, how much money will Homer have to pay to repay the debt together with the interest?

7. (16 points) Consider a spring with mass $m = 1$, and spring constant $k = 5$.

a) Suppose that the damping constant is $\mu = 2$. Find the displacement of the spring as a function of time if the spring is started from spring-mass equilibrium with a velocity of 4.

b) What value of μ makes the spring critically damped?