Math 211

Exam # 1

October 5, 1999

Part 2

Instructions: Part 2 of Exam #1 is an open book, open notes, untimed, take home exam. It is due in the Mathematics Department Office, HB 220, by 4:00 PM on Friday, October 8. You may not discuss the exam with your fellow students. If you have any questions, consult with one of the instructors for the course.

Write out and sign the pledge on your submitted solutions. Be sure to put your name on your submission. In addition put the name of your instructor at the top of the first page of your submission.

Please give reasons for all of your answers. Remember that some reasons are better than others. For example, it is better to refer to a theorem stated in class and/or in the books than to say "the computer printout shows that ...". Of course sometimes the computer printout is all you have.

In answering these questions you are not limited to the use of MATLAB, although it will be useful. You should use any combination of the analytic, qualitative, or numerical methods you have learned.

Your answers should consist of complete sentences organized into coherent paragraphs. This does not mean that they have to be long. Brevity is frequently a sign of understanding. MATLAB graphics should be numbered and referred to by that number. Any graphic which is not referred to will not be counted as part of your submission.

You should consider using MULTIGRAF to put several MATLAB figures on one page and save a tree or two.

Notice that the points on the questions add to 40.

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1. (12 points) Particular solutions to differential equations can sometimes be discovered by observing graphs of solutions. The simplest example is a *straight line solution*, which has a graph which is a straight line. `dfield5` is useful for making such discoveries. We will consider the example $y' = \sin(y-t)+1$

a) Enter the equation into `dfield5` and plot a few solutions to see if you can find evidence of straight line solutions. Submit a `dfield5` figure as your evidence, and state a conjecture as to the location and slopes of the straightline solutions.

b) A solution whose graph is a straight line has the form $y = at + b$, where a and b are constants. Substitute $y = at + b$ into the differential equation and determine what a and b have to be in order that $y = at + b$ is a solution. Find all straight line solutions.

c) On a fresh `dfield5` Display Window (use Edit –> Erase all solutions) have `dfield5` draw some straightline solutions by entering the appropriate initial data.

2. (18 points) A population of fish in a lake is governed by the logistic equation

$$P' = 0.5P(1 - P/10).$$

Population is measured in units of 100,000 fish, and time is in years. The people living near the lake start fishing, and in the process remove $AP/(1+P)$ fish per year, where A is a constant. Hence the model becomes

$$P' = 0.5P(1 - P/10) - \frac{AP}{1+P}.$$

Your job is to discover all posible long term outcomes for the fish population for three values of A, $A = 0.3$, $A = 0.9$, and $A = 1.6$. For each of these cases do the following

a) Enter the equation into `dfield5` and plot enough solutions so that you can make a conjecture as to all possible outcomes. In the process you should conjecture as to the locations of the equilibrium points, and the type of each (i.e., are they stable or unstable?). Carefully state which initial conditions lead to which outcome. You should examine the populations over several time ranges to be sure that you do not miss interesting phenomena.

b) Verify your conjecture by computing the equilibrium points *algebraically* and deciding if they are stable or unstable. We are only interested in realistic populations, so ignore possibilities that are negative or complex.

3. (10 points) In this problem you are to examine possible outcomes of two species which are competing for resources. The model to be used is

$$\begin{aligned} x' &= (1 - x - y)x \\ y' &= (A - Bx - Cy)y, \end{aligned}$$

where $x(t)$ and $y(t)$ are the populations of the two species. If you choose the menu item Gallery –> Competing Species in the `pplane5` Setup window, this system will be entered into `pplane5`. There are two cases to be considered

a) $A = 4$, $B = 2$, and $C = 7$. (This is the default.)

b) $A = 4$, $B = 2$, and $C = 3$.

For each of these cases plot enough solutions so that you can conjecture the long term behavior of all solutions. It may be necessary to change the size of the display window. For each case write a short paragraph describing your conjectures. Add `pplane5` plots as you feel necessary to provide evidence.