Math 211

Exam # 2

November 13, 1997

Part 2

Instructions: Part 2 of Exam #2 is an open book, open notes, untimed, take home exam. It is due in the Mathematics Department Office by 3:30 PM on Monday, November 17. You may not consult with your fellow students about the exam. If you have any questions, consult with one of the instructors for the course.

Write out and sign the pledge on your submitted solutions. Be sure to put your name on your submission. In addition put the name of your instructor at the top of the first page of your submission.

Please give reasons for all of your answers. Remember that some reasons are better than others. For example, it is better to refer to a theorem stated in class and/or in the books than to say "the computer printout shows that ...". Of course sometimes the computer printout is all you have.

In answering these questions you are not limited to the use of MATLAB, although it will be useful. You should use any combination of the analytic, qualitative, or numerical methods you have learned.

Probably most of you will want to use a MATLAB diary file in answering these questions. The ideal way to prepare your submission is to use an editor to insert your comments into the diary file. If you use the editor to leave enough room, equations can be entered by hand. If you do not want to use an editor it is still your responsibility to organize your work in a readable and orderly manner. Your comments should consist of complete sentences organized into cohesive paragraphs. This does not mean that they have to be lengthy. In fact brevity is frequently a sign of understanding. If you include MATLAB graphics they should be numbered and referred to by that number. Any graphic which is not referred to will not be counted as part of your submission.

The matrices $\mathbf{A}$, $\mathbf{B}$, and the vector $\mathbf{b}$ can be loaded into your MATLAB workspace by executing the m-file `exam2data.m`. Those of you who are working at an owlnet workstation will find that this m-file is already loaded. Those of you who are working in other circumstances should down load the file from the Math 211 web site.

To show that you really understand the matrix algebra, and not just how to use fancy MATLAB commands, you should use `rref` to solve systems of equations, you should NOT use `null` to find nullspaces, and you should NOT use `eig` to compute eigenvectors. You can use `eig` to compute eigenvalues, however.

For the first four problems we will consider the matrix

$$\mathbf{A} = \begin{pmatrix} 4 & -2 & -4 & -3 \\ -1 & 3 & 8 & -3 \\ 2 & -2 & -5 & 0 \\ 2 & -2 & -4 & -1 \end{pmatrix}.$$

1. (5 points) Find the eigenvalues of $\mathbf{A}$.

2. (5 points) You should have discovered in the first problem that there are three distinct eigenvalues, and that one of the eigenvalues of $\mathbf{A}$ has multiplicity 2 (i.e., it is repeated). Denoting this eigenvalue by λ, find a basis for the nullspace of $\mathbf{A} - \lambda\mathbf{I}$.

3. (5 points) Find four solutions to the system $\mathbf{y}' = \mathbf{A}\mathbf{y}$ which are linearly independent at $t = 0$. Give a reason why your solutions are linearly independent. You will get 4 points if you only find three such solutions.

4. (5 points) Find the general solution to the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$ where

$$\mathbf{b} = \begin{pmatrix} 7 \\ 2 \\ 2 \\ 3 \end{pmatrix}.$$

For the next two problems we will use the matrix

$$\mathbf{B} = \begin{pmatrix} 3 & 0 & 4 \\ 6 & -3 & 8 \\ 2 & -2 & 3 \end{pmatrix}.$$

5. (5 points) Find the general solution to the system $\mathbf{y}' = \mathbf{B}\mathbf{y}$.

6. (5 points) Find the solution to the equation $\mathbf{y}' = \mathbf{B}\mathbf{y}$ with initial condition

$$\mathbf{y}(0) = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}.$$

Sometimes a harmonic oscillator is forced, i.e., there is an external force which affects the motion. In this case the equation becomes

$$my'' + k_d y' + k_s y = F(t),$$

where $F(t)$ is the external forcing term. We will examine the case where $F(t) = 10\cos(3.5t)$.

7. (2 points) Show that the second order equation is equivalent to the system

$$\begin{aligned} y' &= v \\ v' &= (-k_s y - k_d v + F(t))\,/m \end{aligned}$$

8. (4 points) In the case when $m = 1$, $k_d = 4$, and $k_s = 16$, use `ode45` to solve the system in Problem 7 for the three sets of initial conditions $y(0) = 3, 10, 20$, with $v(0) = 0$ in each case. Solve over the time interval [0,10]. For each case plot the solution in the phase plane. Submit the three phase plane plots.

9. (4 points) For the three solutions in Problem 8, plot the displacement y against time on one graph. (Remember that `ode45` chooses its own time steps, and they are likely to be different for the three cases. Therefore it might be prudent to give the computed time vector different labels in the three cases.)