

Math 211
Final Exam
Spring 2004

This test has 11 pages. There are 10 problems with a total point value of **100**. Please make sure your test is complete.

This is a closed book, closed notes exam.

You have **3 hours** to complete this exam. Make sure to show your work and justify your arguments.

Calculator policy: You may use calculators to evaluate standard functions on floating point numbers (like $\sqrt{3.12}$, $\ln(35/7)$, or $\sin(\pi/17)$). You may not use symbolic operations, numerical integration, or any graphing functions.

Print your name:

Write out and sign the pledge:

1	2	3	4	5	6	7	8	9	10	T

1. (10p) Suppose that

$$y(x) = \frac{1}{x} + \frac{x}{c}$$

is the solution of the initial value problem

$$y' = 1 - \frac{y}{x}, \quad y(1) = y_0.$$

Find y_0 .

2. (10p) Consider the differential equation

$$y' = \frac{x^2 e^{-y}}{9}.$$

(a) Find the general solution.

(b) Find a particular solution with $y(0) = 0$ and identify its interval of existence.

3. (10p) Find a solution to the initial value problem

$$xy' = \frac{\cos x}{x} - 2y, \quad y(\pi/2) = 0.$$

4. (10p) Homer opens an account that pays an annual rate of 3% compounded continuously with an initial investment of \$5000. After that, he deposits \$1000 per year. Assuming that no withdrawals are made, how much will be in the account at the end of a 10-year period?

5. (10p) Consider the vectors $u = \begin{bmatrix} 1 \\ c \\ c \end{bmatrix}$ and $v = \begin{bmatrix} c \\ 1 \\ c \end{bmatrix}$. Find c so that $w = \begin{bmatrix} c \\ c \\ 1 \end{bmatrix}$ is a linear combination of u and v . Also, write w as a linear combination of u and v .

6. (10p) Find the nullspace for the following matrices:

(a) $A = [0 \ 0 \ 1 \ 0]$.

(b) $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

7. (10p) Find a fundamental set of solutions for the following system:

$$\begin{aligned}x' &= 4x + y \\y' &= -9x - 2y.\end{aligned}$$

Also, find the solution to the initial value problem if $x(0) = 1$ and $y(0) = 2$.

8. (10p) Consider the periodically forced harmonic ordinary differential equation

$$y'' + 5y' + 4y = 2 \sin 3t.$$

- (a) Find a fundamental set of solutions of the associated homogeneous equation.
- (b) Find the steady-state solution and determine its amplitude.

9. (10p) Consider the system of ordinary differential equations

$$\begin{aligned}x' &= 2x - 2xy \\y' &= 2y - x^2 + y^2.\end{aligned}$$

- (a) Find all equilibrium points.
- (b) Compute the Jacobian matrix.
- (c) Classify the equilibrium points (as sinks, sources or saddles).

10. (10p) Consider the system of ordinary differential equations

$$\begin{aligned}x' &= -1 - y + x^2 \\y' &= x + xy.\end{aligned}$$

(a) Show that the unit circle $x^2 + y^2 = 1$ is invariant.

(b) Is there a solution which is not constant and

$$\lim_{t \rightarrow \infty} x(t) = 0 \text{ and } \lim_{t \rightarrow -\infty} x(t) = 0?$$