Math 211
Second Midterm
March 30, 2004

Make sure to show your work and justify your arguments.

Calculator policy: You may NOT use a calculator on this exam. The reasons are twofold: first, the calculations are not hard and you will be able to do them in your head. Second, some calculators are equipped with programs which evaluate determinants and do matrix operations.

1)(15p) Find the nullspace of the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & 0 \\ 3 & 8 & -1 \end{bmatrix}.$$

2)(15p) Consider the following system in unknowns x and y:

$$\begin{aligned} ax + y &= 1 \\ x + by &= 2b \end{aligned}$$

For which values of a and b does the system have a unique solution, no solution or infinitely many solutions?

3)(15p) Find k such that $w = \begin{bmatrix} 2 \\ 3 \\ k \end{bmatrix}$ is in the span of u and v, where

$$u = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} \qquad v = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}.$$

4) Consider the system of ODE's

$$\begin{aligned} x' &= -x - 2y \\ y' &= x - 4y \end{aligned}$$

a) (8p) Find a fundamental set of solutions.
b) (7p) Sketch the phase plane picture for this system and classify the equilibrium point.

EXAM CONTINUES ON BACK PAGE

5) Consider the system of ODE's

$$\begin{aligned} x' &= x + y^2 \\ y' &= -x + y + y^2 \end{aligned}$$

a)(5p) Verify that $x = e^{2t}$, $y = e^t$ is a solution.
b)(5p) Verify that $x = e^{2t}$, $y = -e^t$ is a solution.
c)(10p) Consider the solution to the system with initial conditions $x(0) = -1$, $y(0) = 0$. Is there a time T such that $x(T) > y^2(T)$? Explain.

6)(5p) Find the nullspace of the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

7)(15p) Consider the system of ODE's $x' = Ax$ where

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & -2 & -4 \\ 1 & 1 & 1 \end{bmatrix}.$$

We know that $\lambda_1 = -1 + 2i$ is an eigenvalue with corresponding eigenvector

$$v = \begin{bmatrix} -1 \\ -1 + 2i \\ 1 \end{bmatrix}.$$

(You do not have to verify this.) Give a fundamental set of solutions for the system. (The following fact might help in your computations: the sum of the eigenvalues of a matrix is equal to the trace of the matrix.)