For an $n \times n$ matrix A, the following are equivalent:

1. A is nonsingular.
2. A is invertible.
3. $\det(A) \neq 0$.
4. $A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\mathbf{0}$.
5. A is row equivalent to I.
6. $A\mathbf{x} = \mathbf{b}$ has a unique solution for any $\mathbf{b} \in \mathbb{R}^n$.
7. When A is reduced to row echelon form, there are no free columns/variables.
8. When A is reduced to row echelon form, there are nonzero entries along the diagonal.
9. Null(A)=$\mathbf{0}$.
10. The column vectors of A are a linearly independent set.
11. The column vectors of A form a basis for $\mathbb{R}^n$.

For an $n \times n$ matrix A, the following are equivalent:

1. A is singular.
2. A is not invertible.
3. $\det(A) = 0$.
4. $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution (in fact, it has infinitely many nontrivial solutions).
5. A is not row equivalent to I.
6. There exists some $\mathbf{b} \in \mathbb{R}^n$ such that $A\mathbf{x} = \mathbf{b}$ has no solution.
7. When A is reduced to row echelon form, there are free columns/variables.
8. When A is reduced to row echelon form, there is at least one zero on the diagonal.
9. $\mathrm{Null}(A)$ is nontrivial.
10. The column vectors of A are a linearly dependent set.
11. The column vectors of A do not form a basis for $\mathbb{R}^n$.