Math 102 Practice #1 Solutions for Exam 2

Spring 2008

1. Find the third degree Taylor polynomial for $f(x) = \tan x$ at $a = 0$.

$$x + \frac{1}{3}x^3$$

2. Determine whether the infinite series $\displaystyle\sum_{n=2}^{\infty} \frac{5 \cdot 3^n}{4^{n+1}}$ converges or diverges. If it converges, find its sum.

It converges to $\dfrac{45}{16}$.

3. Determine whether the following infinite series converge or diverge.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{5n + 19}{n^2 + 7n + 10}$.

   Diverges (try comparison test)

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$.

   Diverges (try integral test)

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2(\frac{1}{n})}{\sqrt{n^3 + 1}}$.

   Converges (try comparison test)

4. Use the Taylor series for $\sin x$ to find a power series representation for $\dfrac{\sin(x^2)}{x}$.

$$x - \frac{x^5}{3!} + \frac{x^9}{5!} - \frac{x^{13}}{7!} + \dots$$

5. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4+2}$. Does it converge? Does it converge absolutely?

Converges absolutely (try AST and comparison)

6. Find the interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n(x-1)^n}{\sqrt[3]{n+3}}$.

$$\left(\frac{2}{3}, \frac{4}{3}\right]$$