

Finite element method

The meshing method

steps of the finite element method^[1]

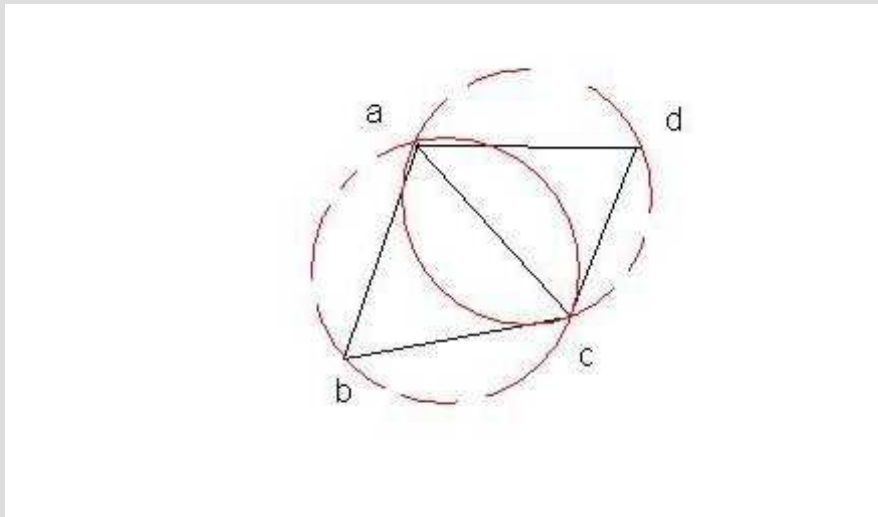
- discretization or subdivision of the domain
- selection of the interpolation functions
- formulation of the system of equations
- solution of the system of equations

[1] "The finite element method in Electromagnetics" Jianming Jin

elements

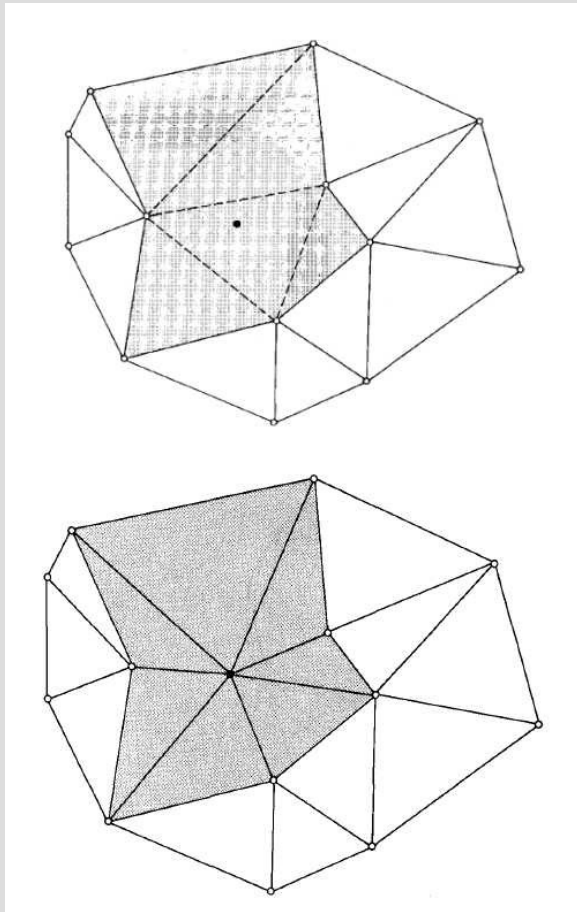
- elements can be any shape
- triangle/tetrahedral elements follow samples very well
- requirements of elements
 - equilateral elements preferred
error is inversely proportional to the sine of the smallest inner angle
 - the smaller the element, the more accurate the solution

Delaunay Triangulation



- Delaunay triangulations maximize the minimum angle of all the angles of the triangles in the triangulation^[1]
- circumcircle formed by triangular contains no other vertices except the three points

Random incremental algorithm^[1]



- initial boundary triangular contains all interested area
- add random point
- find all triangles whose circumcircle contains new point
- remove those triangles and create new by connecting old vertices with new one

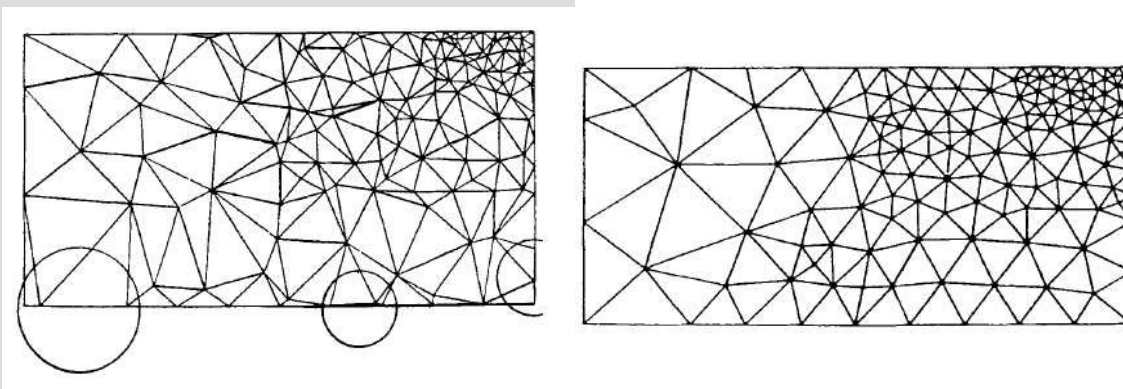
[1]James C.C etc. An approach to automatic three-dimensional finite element mesh generation.

Laplacian smoothing^[1]

- improves Delaunay triangulation
- E_1, E_2, \dots, E_k share vertex z^* , E_i 's remaining vertices are z_1, z_2, \dots, z_k , define new z^* as

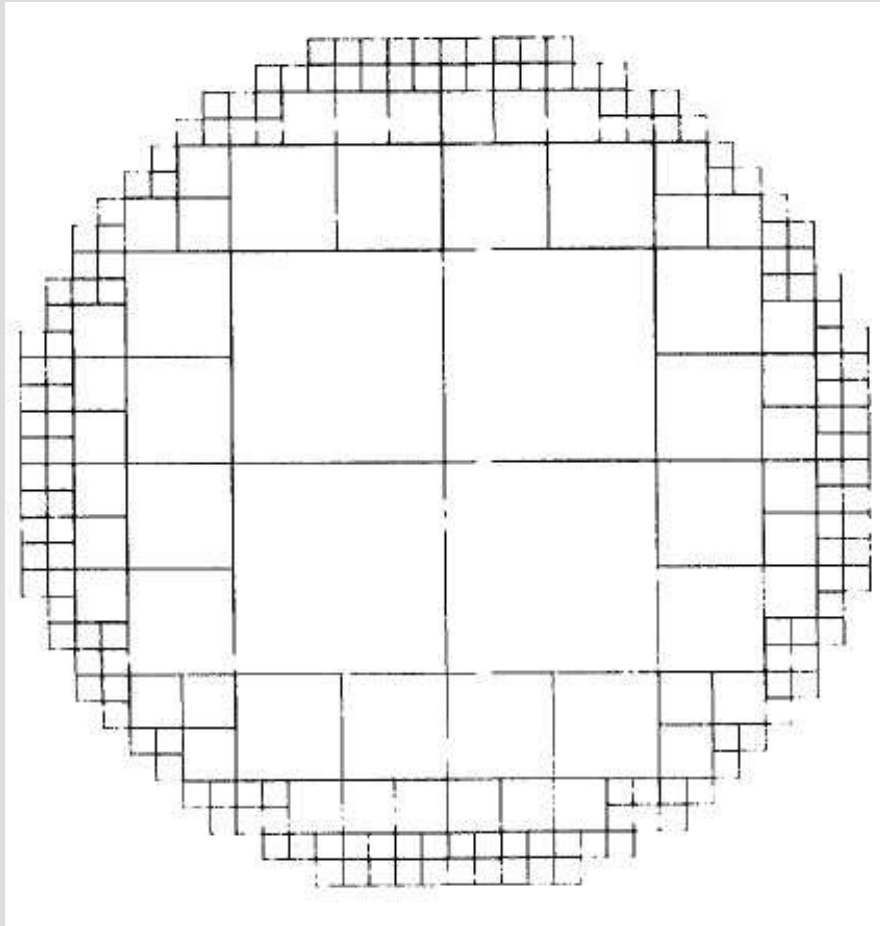
$$z^* = (z_1 + z_2 + \dots + z_k) / k$$

- if z^* does not satisfy Delaunay triangulation, retriangulate



[1]David A.Field, Communication in applied numerical method, vol. 4, 709-712(1988)

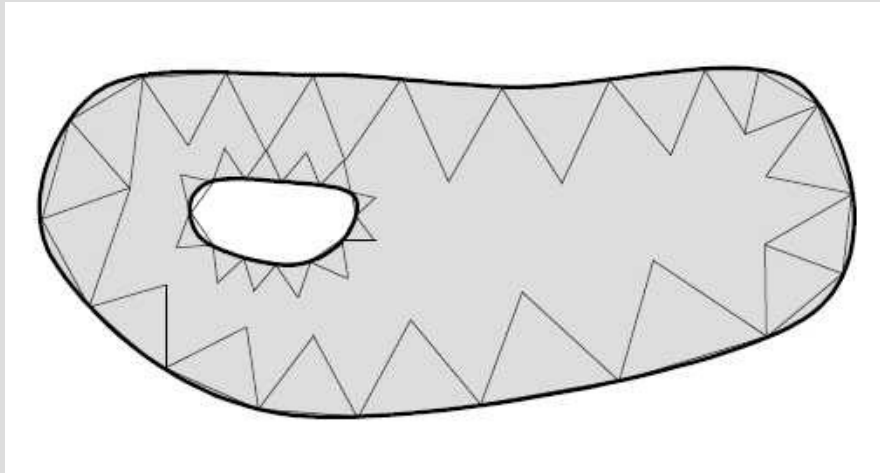
Quadtree/Octree method^[1]



[1]Mark A.Y etc. Automatic three-dimensional mesh generation by the modified-octree technique

- rectangle elements
- steps
 - define a square encompasses entire sample
 - divide the square into four quadrants, and for each quadrant
 - test if it is inside, outside, partially inside
 - divide partial inside quadrant furthermore

advancing front



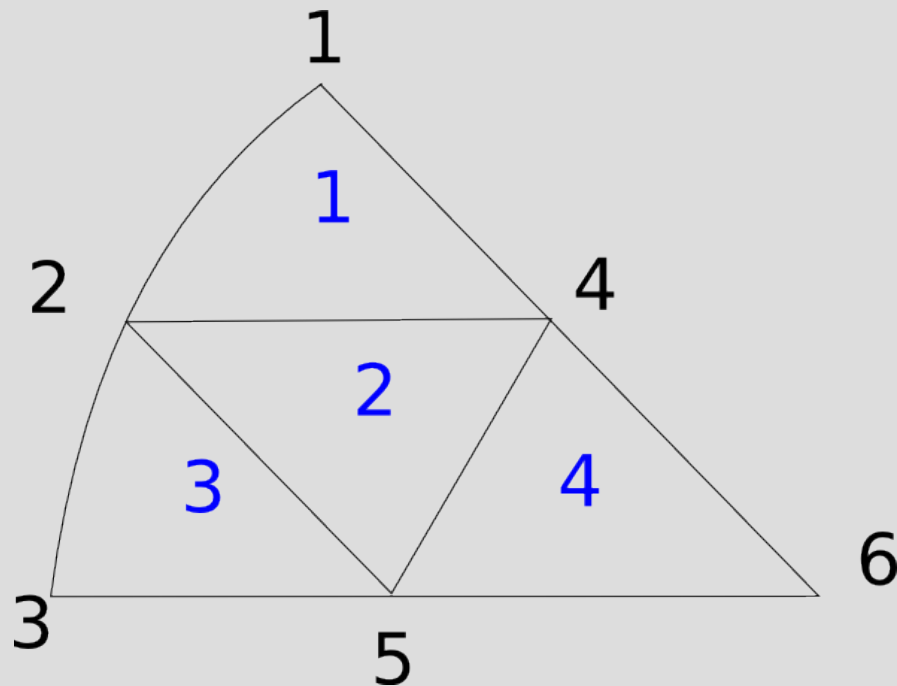
Steven J. Owen, A survey of Unstructured Mesh generation Technology

- meshing from the boundary
- algorithm
 - initialize front on boundary
 - choose an edge to be deleted
 - select a new point
 - insert new point, triangular
 - stop when front is empty

complexity

- Delaunay $O(n \log(n))$
- advancing front $O(n^{3/2})$
- quadtree/octree $O(\log(n))$

use meshing



- numbering of vertices, elements
- local numbering to global numbering