

Thomson Scattering

(Scattering of EM waves by free particles)

Assume a plane wave with wave vector \underline{k}_0 and polarization (unit) vector $\underline{\epsilon}_0$:

$$\underline{E}(\underline{x}, t) = \underline{\epsilon}_0 E_0 e^{i(\underline{k}_0 \cdot \underline{x} - \omega t)}$$

For a free particle, this wave field produces an acceleration

$$\underline{\dot{v}}(t) = \frac{qE}{m} = \underline{\epsilon}_0 \frac{q}{m} E_0 e^{i(\underline{k}_0 \cdot \underline{x} - \omega t)} \quad (14.121)$$

(assuming $\beta = v/c \ll 1$).

Like any acceleration, this acceleration produces radiation, at the same frequency ω but in arbitrary directions (averaged over many scatterers). It is similar to dipole scattering, discussed in chapt. 10 and L25.

($\beta \rightarrow 0$)

Eqn. (14.38) gives the power radiated per unit solid angle

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} |\hat{n} \times (\hat{n} \times \underline{\dot{v}})|^2 \quad (14.20)$$

$$\rightarrow \frac{q^2}{4\pi c^3} |\underline{\epsilon} \cdot \underline{\dot{v}}|^2 \quad \text{for polarization state } \underline{\epsilon}.$$

(14.120)

Substituting (14.121) into (14.20) gives

$$\frac{dP}{d\Omega} = \frac{c}{8\pi} |E_0|^2 \left(\frac{e^2}{mc^2}\right)^2 |\underline{\epsilon} \cdot \underline{\epsilon}_0|^2 \quad (14.122)$$

But $\frac{c}{8\pi} |E_0|^2 =$ energy flux of incident wave.

As in chapt. 10 (L25), we define a differential scattering cross section by

$$\frac{d\sigma}{d\Omega} = \frac{\text{scattered power per unit solid angle}}{\text{incident power per unit area}}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2 |\underline{\epsilon} \cdot \underline{\epsilon}_0|^2 \quad (14.124)$$

For unpolarized incident radiation (randomly distributed $\underline{\epsilon}_0$) this can be averaged over $\underline{\epsilon}_0$ to obtain

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2 \frac{1}{2} (1 + \cos^2\theta) \quad (14.125)$$

the Thomson formula, where $\theta =$ angle between \hat{n} and \underline{k}_0 . Integrating over solid angle $d\Omega$ gives the Thomson cross section

$$\sigma_T = \frac{8\pi}{3} \left(\frac{e^2}{mc^2}\right)^2 \quad (14.126)$$

which is $\frac{8}{3}$ times the geometric cross section of a sphere with radius = classical electron radius.

The Thomson result is good for small-angle scattering, where the momentum of the scattered photon $h\nu/c$ can be neglected. For $\theta \neq 0$ there is a correction, due to Compton, to include photon momentum:

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2 \left(\frac{k'}{k}\right)^2 |\underline{\epsilon} \cdot \underline{\epsilon}_0|^2 \quad (14.127)$$

where

$$\frac{k'}{k} \approx \frac{1}{1 + \frac{h\nu}{mc^2}(1 - \cos\theta)}$$

neglecting the electron spin. Here k' = wave number of scattered radiation and $k (= |\underline{k}_0|)$ is that of the incident wave. They differ because of the recoil of the electron ($\Rightarrow k' < k$).

To include electron spin we need a quantum theory and the resultant Klein-Nishina formula (Jackson, p. 697).

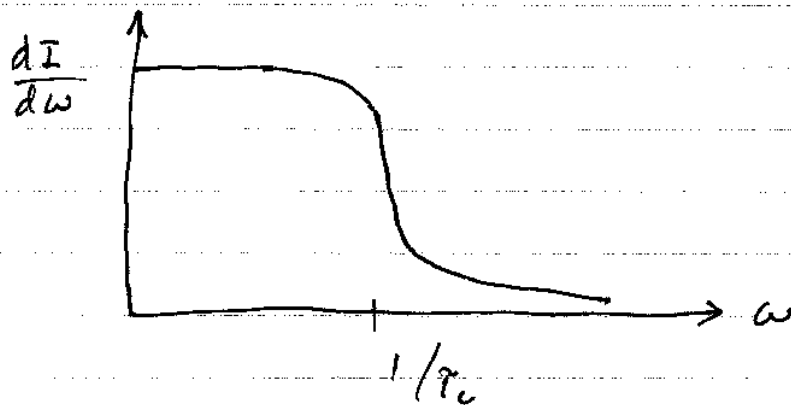
In all cases the cross-section $\propto 1/m^2$, so Thomson/Compton scattering is most important for electrons (or positrons).

Bremsstrahlung (Chapt. 15)

Bremsstrahlung (German for braking radiation) is EM radiation produced by acceleration of charged particles during collisions in a medium. It is most important for relativistic particles. (For non-rel. particles, most of the energy loss goes into heating the medium.)

At low frequencies, Brem. is electric dipole radiation polarized in the plane of \hat{n} and $\Delta\beta$.

The freq. spectrum is generally flat up to a frequency $\sim 1/\tau_c$ where $\tau_c = \text{duration}$ of a collision (not interval between collisions), then it falls off rapidly for $\omega > 1/\tau_c$:



(End of Chapt. 15)

Radiation Reaction (Chapt. 16)

This is the effect of radiation on the dynamics of the particles that produce it. Jackson argues (§ 16.1) that this effect is negligible for dynamic times scales

$$\tau \gg T_{rr} \equiv \frac{2}{3} \frac{e^2}{mc^3} \quad (16.3)$$

For electrons, $T_{rr} \sim 6.3 \times 10^{-24}$ sec (= $2/3$ times the time required for light to cross one classical electron radius). It is even smaller for more massive particles.

There is an interesting exception to the Jackson rule for synchrotron-radiating electrons in strong magnetic fields. Because electrons lose kinetic energy to the radiation, their orbits are not exactly circles, but inward spirals.

How much does the orbit differ from a circle?

Equivalently, how many orbits does it take before an electron radiates away a significant fraction of its kinetic energy?

From L37, the power radiated by a single electron is

$$P = \frac{2}{3} \frac{e^4 B^2}{m_e^2 c^3} \gamma^2 \quad (\times 14.18)$$

So the electron, with energy $\gamma m_e c^2$, has a radiative lifetime

$$\begin{aligned} \tau_s &= \frac{E}{P} = \frac{3}{2} \frac{\gamma m_e^2 m_e^2 c^3}{e^4 B^2 \gamma^2} = \frac{3 m_e^3 c^5}{2 e^4 B^2 \gamma} \\ &= \frac{3 m_e^3 c^3}{2 e^2 \omega_B^2 \gamma} \quad (\times 14.19) \end{aligned}$$

where $\omega_B = \frac{eB}{m_e c} =$ non-relativistic gyrofrequency

$$\sim 2 \times 10^7 \frac{\text{rad}}{\text{s}} \left(\frac{B}{1 \text{ G}} \right) \quad (\times 14.20)$$

(= γ x gyrofrequency of relativistic particle)

In terms of τ_{rr} (16.3), (x14.19) can be written

$$\tau_s = \frac{1}{\gamma \omega_B^2 \tau_{rr}} \quad (\times 14.21)$$

Since τ_{rr} is very small, this would suggest that τ_s is very large, but we should check this because ω_B can get very large for large B (x14.20).

For $B \sim 1 \text{ G}$ (e.g., the solar surface, or the synchrotron radiation belts of Jupiter), (x14.19) gives

$$T_s \sim 5 \times 10^7 \text{ s} \left(\frac{10}{\gamma}\right) \sim 600 \text{ days} \left(\frac{10}{\gamma}\right)$$

$$\gggg \frac{2\pi\gamma}{\omega_B} \sim 3.5 \times 10^{-6} \text{ s} \left(\frac{\gamma}{10}\right)$$

so an electron can gyrate lots of times ($N \sim 10^{12}$) before losing much energy. The assumption of a circular orbit is a very good one.

On the other hand, a radiative lifetime $\sim 2 \text{ yrs}$ is not absurdly long in the context of Jovian magnetospheric dynamics.

Relativistic electrons are supplied to the Jovian synchrotron radiation belt ($r \sim 2 R_J$) by an inward transport process with a timescale of several months if not years. This provides not only inward transport but also acceleration (by the "betatron" process with $\gamma^2 \propto B \propto r^{-3}$, resulting in radiation power $P \propto r^2 B^2$ (x14.18) $\propto r^{-9}$.)

When the electrons reach a field strength $B \sim 1 \text{ G}$ ($r \sim 2 R_J$), it is like a brick wall. They radiate away their energy before they can go any closer. (Good news for people living on Jupiter.)

Another example: In a modern lab. synchrotron, you might have $B \sim 0.5 \text{ T} = 5 \times 10^3 \text{ G}$. Then (from (x14.20)) $\omega_B \sim 10^{11} \text{ /s}$ and (from (x14.19))

$$\tau_s \sim \frac{20 \text{ s}}{\gamma} \sim (10^{-3} \text{ s}) \left(\frac{2 \times 10^4}{\gamma} \right)$$

which is \gg (but not \gggg) $\frac{2\pi\delta}{\omega_B} \sim 10^{-6} \text{ s} \left(\frac{\gamma}{2 \times 10^4} \right)$.
 ($\gamma = 2 \times 10^4 \leftrightarrow E = 10 \text{ GeV}$)

If you pull the plug, the electrons will radiate their energy away in ~ 1000 gyrations $\sim 1 \text{ ns}$.

A more dramatic example is provided by pulsar magnetospheres, where $B \sim 10^{12} \text{ G}$ is inferred. This $\Rightarrow \omega_B \sim 2 \times 10^{19} \text{ /s}$ (from (x14.20)) and, from (x14.21),

$$\tau_s \sim 2 \times 10^{-22} \text{ s} \left(\frac{2 \times 10^6}{\gamma} \right)$$

$$\ll \frac{2\pi\delta}{\omega_B} \sim 6 \times 10^{-13} \text{ s} \left(\frac{\gamma}{2 \times 10^6} \right)$$

$$(\gamma = 2 \times 10^6 \leftrightarrow E = 1 \text{ TeV})$$

so an electron of this energy cannot complete a single gyration before its energy goes poof!

and positrons
 This is why ultrarelativistic electrons, in pulsar magnetospheres are unanimously assumed to be aligned almost perfectly with $\pm \underline{B}$. Any deviation would be radiated away instantly.