Following Jackson, we now switch from SI units to Gaussian units. It is conventional to use SI units in describing macroscopic phenomena (chapters 1-10) but Gaussian units in describing electrodynamic interactions of individual charged particles (chapters 11-16). Here are the relevant equations in both sets of units:

SI (mks) Units	Gaussian (cgs) units	
$\nabla\cdot\mathbf{E}=\rho/\varepsilon_o$ *or* $\nabla\cdot\mathbf{D}=\rho$	$\nabla\cdot\mathbf{E}=4\pi\rho$ *or* $\nabla\cdot\mathbf{D}=4\pi\rho$	(X11.1)
$\nabla\times\mathbf{B}=\mu_o\mathbf{J}+\frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}$ *or* $\nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}$	$\nabla\times\mathbf{B}=\frac{4\pi}{c}\mathbf{J}+\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}$ *or* $\nabla\times\mathbf{H}=\frac{4\pi}{c}\mathbf{J}+\frac{1}{c}\frac{\partial\mathbf{D}}{\partial t}$	(X11.2)
$\nabla\cdot\mathbf{B}=0$	$\nabla\cdot\mathbf{B}=0$	(X11.3)
$\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$	$\nabla\times\mathbf{E}+\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}=0$	(X11.4)
$\mathbf{D}=\varepsilon_o\mathbf{E}+\mathbf{P}$	$\mathbf{D}=\mathbf{E}+4\pi\mathbf{P}$	(X11.5)
$\mathbf{H}=\frac{\mathbf{B}}{\mu_o}-\mathbf{M}$	$\mathbf{H}=\mathbf{B}-4\pi\mathbf{M}$	(X11.6)
$\mathbf{F}=q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$	$\mathbf{F}=q\left(\mathbf{E}+\frac{\mathbf{v}\times\mathbf{B}}{c}\right)$	(X11.7)

For more detail, see Jackson's appendix.

Units for basic electromagnetic quantities:

Quantity	SI unit	= ____ ×	Gaussian unit
Charge	Coulomb	$= 3\times10^9 \times$	statcoulomb
Current	Ampere	$= 3\times10^9 \times$	statampere
Electric Field	Volt/m	$= \frac{1}{3}\times10^{-4}\times$	Statvolt/cm
Magnetic Induction	Tesla	$= 10^4 \times$	Gauss

See Jackson appendix for more.

CHAPTER 11. SPECIAL THEORY OF RELATIVITY

Precise description of a physical phenomenon requires specification of a reference frame, i.e., how fast (and in what direction) the observer is moving.

Consider how a measurement is made. Construct an xyz grid of identical meter sticks. There is an observer sitting at each grid point, with his/her (x,y,z) coordinates printed on his/her shirt, and with his/her own clock. Each clock keeps perfect time, and all clocks were synchronized when they were given to the observers, who then moved to their respective grid points very slowly. Once they get there, they never leave.

If one observer sees something happen at his/her location, he/she records the event and time and mails a letter to the chief observer, who is at the origin. Each observer's stationery has his/her (x,y,z) coordinates printed on it. So each event is characterized by a set of four numbers (x,y,z,t). The observers mind their own business – they never record events that occur outside their grid sector.

A reference frame consists of one such set of observers, each with his/her own (x,y,z) and his/her own clock.

Assume that many such reference frames exist, moving relative to one another. (Disregard the mechanical problem of one grid of meter sticks moving through another.) The same events can be measured in every frame. The set of numbers (x,y,z,t) for a given event will depend on which frame it is measured in.

An <u>inertial</u> reference frame is one in which a body that is subject to no external force moves at constant velocity. Thus, two inertial frames differ by a constant velocity.

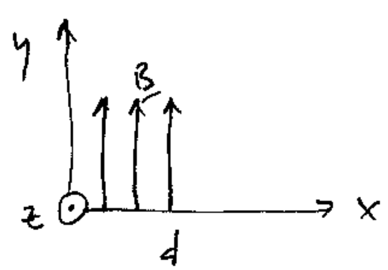
Galilean Transformation Laws for physical quantities measured in two different inertial reference frames (based on everyday experience):

Let $\mathbf{V}$ (= constant) = velocity of the primed system relative to the unprimed system. Then

$$\left.\begin{array}{ll}
\mathbf{x}' = \mathbf{x} - \mathbf{V}t & \text{(a)} \\
t' = t & \text{(b)} \\
\mathbf{v}' = \mathbf{v} - \mathbf{V} & \text{(c)} \\
\mathbf{a}' = d\mathbf{v}'/dt' = d\mathbf{v}/dt = \mathbf{a} & \text{(d)} \\
\mathbf{F}' = \mathbf{F} & \text{(e)} \\
\mathbf{E}' = \mathbf{E} + \mathbf{v}\times\mathbf{B}/c & \text{(f)} \\
\mathbf{B}' = \mathbf{B} & \text{(g)} \\
\rho' = \rho & \text{(h)} \\
\mathbf{J}' = \mathbf{J} - \rho\mathbf{V} & \text{(i)}
\end{array}\right. \qquad \text{(X11.8)}$$

Maxwell's equations are not invariant under Galilean transformations.

<u>Example 1</u>: Suppose, in the unprimed frame, that



$$\left.\begin{array}{lll}
\mathbf{B} = B_0\,\hat{y} & & 0 < x < d \\
\mathbf{B} = 0 & & \text{elsewhere} \\
\mathbf{E} = 0 & & \text{everywhere} \\
\rho = 0 & & \text{everywhere}
\end{array}\right\} \quad (X11.9)$$

[from (X11.1)]

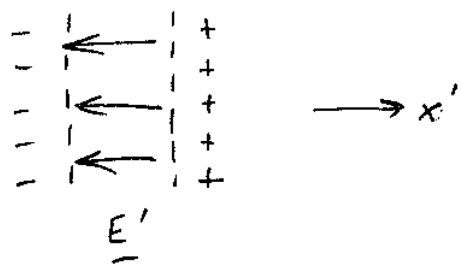
Use a Galilean transformation to view the same situation in a primed frame, moving at $V\hat{z}$ relative to the unprimed frame:

From (X11.8f) and (X11.9),

$$\underline{E}' = \underline{E} + \underline{V} \times \underline{B}/c$$

$$= 0 + \frac{V\hat{z} \times B_0 \hat{y}}{c} = -\frac{VB_0}{c}\hat{x} \qquad 0 < x' < d$$

$$= 0 \quad \text{elsewhere}$$



If we assume that Coulomb's law (X11.1) is valid in the primed frame, then

$$4\pi\rho' = \nabla' \cdot \underline{E}' = -\delta(x')\frac{VB_0}{c} + \delta(x'-d)\frac{VB_0}{c}$$

$$\rho' = \frac{VB_0}{4\pi c}\left[\delta(x'-d) - \delta(x')\right] \qquad \text{(X11.10)}$$

Equations (X11.9) and (X11.10) are inconsistent! Coulomb's law is not invariant under Galilean transformations. Either that equation is valid only in certain preferred frame(s), or the Galilean transformation isn't right.

The most famous example: Maxwell's equations imply that electromagnetic radiation travels at *c* in whatever frame the equations are written in. That's inconsistent with (X11.8c). If $|\mathbf{v}| = c$ in any direction in the unprimed frame, (X11.8c) implies that $|\mathbf{v}'|$ will vary with direction in the primed frame. **Same choice**: either Maxwell's equations are valid only in one reference frame, or the Galilean transformation is wrong.

What to do?

- Maxwell saw this problem immediately.
- He assumed that his equations were valid in only one reference frame.
- Physical interpretation: EM radiation is supported by a physical medium, called the "ether". (Like sound waves need a medium, he figured light waves need a medium.)
- The Michelson-Morley experiment:
  - Looked for dependence of c on direction.
  - Presumably the ether is at rest in an inertial frame. It shouldn't rotate with the Earth. This implies that c should be different for waves moving east vs. west.
  - They found no difference. Had to assume that the ether moved with local matter. (Not satisfactory.)
  - Another possible hypothesis: Matter contracts in the direction of flow through the ether.
- Einstein reconsidered the situation from a more fundamental viewpoint.
  - Galilean transformation implies all velocities depend on reference frame.
  - Michelson-Morley implies speed of light is the same in all frames.
  - Therefore the Galilean transformation must be wrong.

Einstein enunciated the **Principle of Relativity**: <u>The laws of nature are the same in any inertial reference frame</u>. Maxwell's equations then imply that light travels at $c \approx 3.00\times10^{10}$ cm/s in any inertial frame.

This principle, plus some innocuous assumptions, lead to the **Lorentz transformation**.

- Lorentz and others had figured out what transformation was required to make Maxwell's equations invariant.
- They tried to figure out what those transformations meant, but they weren't bold enough to proclaim them as fundamental characteristics of space and time.
- Einstein *was* bold enough.

Derivation of Lorentz Transformation

Assumptions:

1. Primed frame moves at $V\hat{x}$ relative to unprimed. (Defines $\hat{x}$.)
2. Transformation involves no spatial rotations or reflections.
3. Origins coincide at t=0, and clocks synchronized then.
   ($x=y=z=t=0 \rightarrow x'=y'=z'=t'=0$.)
4. Transformation is linear in space and time. (Space-time is homogeneous.)
5. Inverse transformation = transformation with $-V\hat{x}$.
6. Speed of light = c in both frames. ($x = ct \rightarrow x' = ct'$.)
7. Symmetries: y, z, and t scalings are independent of the sign of V.
   Clock synchronization is independent of y,z.

Assumptions 1, 2, 3, and 4 imply the following conditions

$$x' = y' = z' = 0 \Rightarrow x = vt,\ y = z = 0 \qquad (XII.11)$$

$$x = y = z = 0 \Rightarrow x' = -vt',\ y' = z' = 0 \qquad (XII.12)$$

From assumption 4, $y' = ax + by + cz + dt + e$

Assumption 2 $\Rightarrow a = c = 0$, so

$$y' = by + dt + e \qquad (XII.13)$$

Substituting (XII.12) in (XII.13) gives

$$0 = 0 + dt + e \quad \Rightarrow d = e = 0$$

This condition and assumption 7 imply

$$y' = b(V^2)\,y$$

Assumption 5 gives $[b(V^2)]^2 = 1$

which, with assumption (2), implies $b = 1$.

$\therefore \; y' = y$ (XII.14)

A similar argument gives $z' = z$ (XII.15)

Assumption (4) implies $x' = fx + gt + hy + kz + l$
but assumption (2) $\Rightarrow$ $h = k = 0$, $x' = fx + gt + l$
Applying (XII.11) gives $0 = fVt + gt + l$
which $\Rightarrow$ $g = -fV$ and $l = 0$

$$x' = f(x - Vt) \tag{XII.16}$$

Assumption (4) also implies $t' = mx + nt + py + qz + r$.
Synchronization of clocks at origin $\Rightarrow$ $r = 0$.
Assumption 7 $\Rightarrow$ $p = q = 0$.
Thus

$$t' = mx + nt \tag{XII.17}$$

Now apply the crucial assumption (6): $x = ct \Rightarrow x' = ct'$.
Substitute into (XII.16) and (XII.17):

$$ct' = f(ct - Vt) \tag{XII.18}$$

$$t' = mct + nt \tag{XII.19}$$

Substituting (XII.19) in (XII.18) gives

$$c(mc + n) = f(c - V)$$

$$f = \frac{mc + n}{1 - V/c} \tag{XII.20}$$

Now, $x = -ct \Rightarrow x' = -ct'$.
Substituting into (XII.16) and (XII.17) gives

$$-ct' = f(-ct - Vt)$$
$$t' = -mct + nt$$

which combine to

$$f = \frac{n - mc}{1 + V/c} \qquad (XII.21)$$

Equating right sides of (XII.20) and (XII.21) gives

$$m = \frac{-V}{c^2} n \qquad (XII.22)$$

Substituting (XII.22) in (XII.21) gives $\underline{f = n.}$ (XII.23)

Substituting (XII.22) and (XII.23) in (XII.16) gives

$$x' = n(x - Vt) \qquad (XII.24)$$
$$t' = n(t - Vx/c^2) \qquad (XII.25)$$

At the unprimed origin, $t' = nt$. From assumption (7), n shouldn't depend on the sign of V:

$$x' = n(V^2)(x - Vt) \qquad (XII.26)$$
$$t' = n(V^2)(t - Vx/c^2) \qquad (XII.27)$$

The inverse transformation, from assumption 5, is

$$x = n(V^2)(x' + Vt') \qquad (XII.28)$$
$$t = n(V^2)(t' + Vx'/c^2) \qquad (XII.29)$$

Combining (XII.26, 27, 28) gives $x = [n(V^2)]^2 \left[x - Vt + V\left(t - \frac{Vx}{c^2}\right)\right]$

$$[n(V^2)]^2 = \frac{1}{1 - V^2/c^2}$$

Assumption 2 $\Rightarrow$ must take positive root. (XII.28, 29) $\Rightarrow$

$$x' = \frac{x - Vt}{\sqrt{1 - V^2/c^2}} \qquad t' = \frac{t - Vx/c^2}{\sqrt{1 - V^2/c^2}} \qquad \begin{matrix}(XII.30)\\(XII.31)\end{matrix}$$