PHYS 532 HOMEWORK 3 (CHAPTER 6b) (2 pages) Due 2/12/07

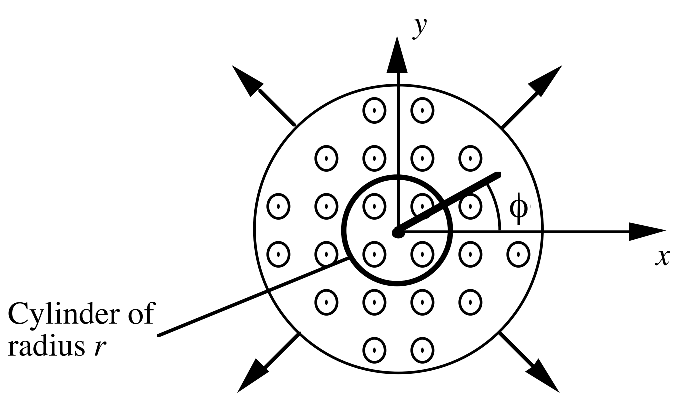
Problem 1. (~Jackson 6.11)

A transverse plane wave is incident normally in vacuum on a perfectly absorbing flat screen.

(*a*) From the law of conservation of linear momentum show that the pressure (called radiation pressure) exerted on the screen is equal to the field energy per unit volume in the wave.

(*b*) In the neighborhood of the Earth, the flux of electromagnetic energy from the Sun is approximately 1.4 kW/m^2. If an interplanetary "sailplane" had a sail of mass per unit area = 1 gm/m^2 and negligible other mass, what would be its approximate acceleration in centimeters per second squared due to the solar radiation pressure? How does this compare with the acceleration due to the solar wind? (For the solar wind, assume a pressure $P = NM\mathrm{v}^2$ where N = number density = 5 cm^{-3}, M = particle mass = 1 amu, and v = flow velocity = 400 km/s.)

Problem 2.



Consider the expanding magnetic cylinder shown above. The magnetic flux inside the cylinder is a constant Φ. The magnetic field is in the z-direction (out of the page). Its strength $B(t)$ is uniform inside the cylinder, but it decreases in time as the cylinder expands. The electric field is in the ϕ-direction. Its strength $E_\phi(r, t)$ is independent of ϕ and may not be negligible compared to $cB(t)$. The problem centers on performing an energy analysis of the region inside a cylindrical volume of fixed radius r that lies inside, and centered on, the expanding volume. Assume $\varepsilon = \varepsilon_o$, $\mu = \mu_o$.

(*a*) Find the strength of the electric field $E_\phi(r, t)$ on the cylinder of radius r, in terms of the function $B(t)$, which is assumed known.

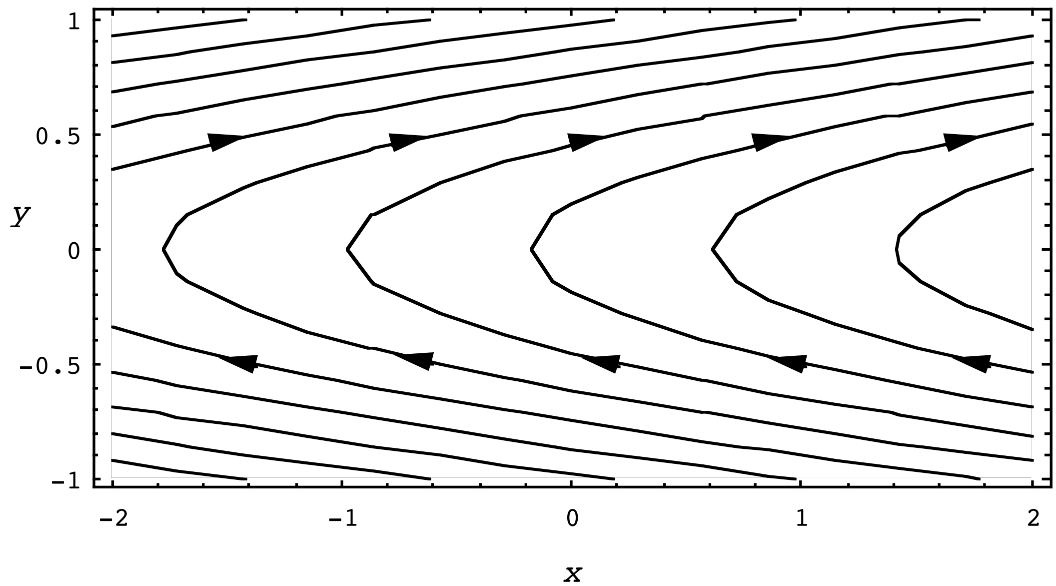
(*b*) What is the total flow of electromagnetic energy outward through the cylindrical surface of radius r, per unit time and per unit length in the z-direction?

(*c*) What is the current density in the cylinder?

(*d*) At what rate is energy fed into particles inside the cylinder, per unit z?

(*e*) What is the rate of change of electromagnetic energy inside the cylinder of radius r, per unit length in z?

Problem 3.



The magnetic field in a conducting fluid is given by

$$\mathbf{B} = B_0 \hat{\mathbf{e}}_y + B_1 \tanh\left(\frac{y}{\Delta}\right) \hat{\mathbf{e}}_x$$

and the electric field $\mathbf{E} = 0$. (Recall that $\tanh(u) \equiv (e^{u} - e^{-u})/(e^{u} + e^{-u})$.)

(a) Find an explicit expression for the Maxwell stress tensor.

(b) Using that stress tensor, calculate the x-component of the force per unit volume exerted by the electromagnetic field on the fluid.