Estimation of parameters for the simulation of foam flow through porous media: Part 1; the dry-out effect

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Estimation of parameters for the simulation of foam flow through porous media: Part 1; the dry-out effect

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Abstract

We perform experimental and modeling studies of foam in porous media. Gas and surfactant solution are co-injected into sand packs at different experimental conditions in two systems. At steady state, saturation of the aqueous phase is shown to be relatively constant over a wide range of foam qualities. In order to obtain an accurate model fit at the transition foam quality, the difference between the foam model parameter $f_{mdry}$ and the transition water saturation $S'_w$ is identified and a method to precisely calculate $S'_w$ is developed. By superimposing contour plots of the transition foam quality and the foam apparent viscosity, one can estimate the reference mobility reduction factor ($f_{mmob}$) and the critical water saturation ($f_{mdry}$) using the STARS™ foam.
model. The parameter $epdry$, which regulates the abruptness of the foam dry-out effect, can be estimated by a transient foam experiment. The effects of surfactant concentration and fluid velocity on foam modeling will be discussed in a following paper based on the findings in the dry-out function in this work.

**Keywords:** foam model; porous media; surfactants; reservoir simulation; fractional flow theory; mobility control

### 1. Introduction

Foam in porous media is defined as a dispersion of gas in liquid such that the liquid phase is continuous and at least some part of the gas phase is made discontinuous by thin liquid films called lamellae. Although these lamellae are thermodynamically unstable, they can be in-situ generated in porous media via various mechanisms. This fact leads to a dynamic balance of liquid lamellae and effective foam propagation through underground formations. Using surfactants as foaming agents to stabilize foams is an effective way to generate strong foams in porous media. These surfactant-stabilized foams were employed as displacing fluids to enhance oil recovery (EOR) in oil reservoirs and to improve sweep efficiency in environmental remediation processes. For example, as mobility control agents, CO$_2$ foams have been successfully tested in the laboratory and through several pilots in the United States. Currently, miscible CO$_2$-EOR processes are tested in heterogeneous carbonate oil...
reservoirs in Abu Dhabi, UAE\textsuperscript{9,10}, and CO\textsubscript{2} foams may also be applicable to enhance sweep efficiency in these reservoirs. Without the use of foams, poor sweep efficiency is caused by low viscosity of CO\textsubscript{2} and heterogeneous conditions between reservoir layers.

Foam reduces the gas mobility by reducing the gas relative permeability ($k_{rg}$) and increasing the effective viscosity of the gas phase ($\mu_g$)\textsuperscript{11}. The mechanism behind this gas relative permeability reduction is the effect of trapped-gas saturation created by foam\textsuperscript{12,13}. For continuous-gas foams, the reduction of gas relative permeability is the only effect of foam. However, for discontinuous-gas foams, the flow resistance of lamellae contributes to an increase in the effective viscosity of the gas phase\textsuperscript{14}. Depending on the foaming agent, the apparent viscosity of surfactant-stabilized foams can be as high as several hundred centipoise\textsuperscript{15}, which is orders of magnitude higher than the viscosity of either gas or water.

In addition to mobility reduction of the gas phase, several important properties of foam transport in porous media have been discovered by a number of researchers. One finding is that there exists a minimum pressure gradient ($\nabla p_{\text{min}}$), or a critical injection velocity in a certain porous medium, to generate strong foam\textsuperscript{3,16}. Trapped bubbles are mobilized above this pressure gradient, and one flowing lamella divides into two lamellae upon a branch point in the flowing channels. The division of liquid lamellae refines foam texture and generates strong foam. Note that lamellae division does not happen without sufficient pressure gradient\textsuperscript{17}. The minimum pressure gradient is dependent on
the permeability of the system and low permeable porous media generally requires high $\nabla p_{\text{min}}$ to generate strong foam $^3$.

Another important finding is the understanding of the limiting capillary pressure ($P_c^*$) for foam stability $^{18}$. When conditions are favorable for generation of finer textured foam, the resulting reduced gas mobility reduces the water saturation, which increases the capillary pressure until the rate of foam coalescence equals foam generation. Meanwhile, typical pressure gradient contours of foam flooding indicate that two regimes exist in which foam flows through porous media: the low-quality regime and the high-quality regime $^{19}$. In the low-quality regime, where the foam contains a high water fraction, bubble trapping and mobilization dominate foam flow $^{20}$; in the high-quality regime, gas bubbles are separated by thin liquid lamellae resulting in a relatively dry foam and its behavior is determined by the limiting capillary pressure and bubble coalescence $^{18, 21}$.

Various foam models have been proposed to simulate foam flow through porous media, among which the most promising ones are the mechanistic methods based on bubble population balance and the semi-empirical methods based on fractional flow theory. The population-balance foam model is a mechanistic method to describe foam flow through porous media $^{12, 22}$. It has received wide attention because it physically describes the generation and decay of foam lamellae and dynamically tracks the mobility of foam. In this model, foam texture (the number of liquid lamellae per unit volume present in porous media) is modeled explicitly with a bubble population-balance equation and gas mobility is
expressed as a function of foam texture, water saturation and other factors.\(^{23}\)

This foam model has successfully matched several laboratory experiments\(^{24-26}\), and is capable of simulating foam behavior in both the high-quality and low-quality regimes upon the modification of net foam generation\(^{27-29}\). However, there are some limitations in this foam model, such as the difficulty in obtaining the modeling parameters, especially at the field scale, and determining the minimum pressure gradient for foam generation\(^{23}\). Additionally, the foam generation term in this model does not distinguish between snap-off and lamella division at various conditions\(^{17,30}\).

In the early studies of the semi-empirical methods, the effect of foam on gas mobility reduction is expressed with a mobility reduction factor (MRF), which reduces the relative permeability of the gas phase depending on surfactant concentration, oil saturation and capillary number.\(^{31}\) Advances to these semi-empirical methods incorporate the mechanistic physics of foam transport to the MRF using the fractional flow theory. According to the fixed-\(P^*\) model,\(^{32}\) the effect of limiting capillary pressure above which foam collapses is included in the foam model in terms of a critical water saturation. This concept was later adopted in a commercial reservoir simulator STARS\(^{TM}\) with a modified expression.\(^{33}\)

Vassenden and Holt proposed a similar model by modifying the gas relative permeability with experimental validation.\(^{34}\) On the other hand, lamella creation and coalescence are balanced by assuming local steady state, and a model modifying gas phase viscosity through foam texture is constructed.\(^{28,29}\) In this model, foam texture is a function of water saturation and pressure gradient if the...
water saturation is above the critical water saturation. According to Darcy’s law, decreasing relatively permeability or increasing viscosity of the gas phase by the same factor results in the same mathematical expression. Therefore, it doesn’t make a difference if a viscosity-based model versus a relative-permeability-based model (STARS™) is utilized. The semi-empirical foam model using the fractional flow theory, or the STARS™ foam model, has been widely used to describe foam flow through porous media in the oil and gas industry. These applications include simulation of surfactant-alternating-gas (SAG) processes and simultaneous injection of surfactant solution and gas, simulation of oil displacement by foam, interpretation of experimental data of co-injection of gas and surfactant solution on the 10 m scale and in Berea cores, and simulation of foam process at reservoir scale from the Snorre field in Norway.

A crucial step for simulating foam flow through oil reservoirs is to estimate the parameters used in the foam model. Because there are many parameters in the STARS™ model, it is important to develop a strategy for quickly estimating them before running the reservoir simulations. In this work, we perform an experimental study of foam flow through porous media and present a feasible way to estimate the parameters in the STARS™ foam model. Specifically, a hybrid contour plot method with the aid of MATLAB is introduced for a process of two-parameter estimation to model the effect of gas fraction on foam apparent viscosity. A transient experiment is used to estimate the parameter $\epsilon_{\text{dry}}$ in the dry-out function.
2. Materials and Methods

2.1 Experimental setup

Two systems at different experimental conditions, denoted as System A and B respectively, are used to collect data in 1-D foam flooding processes. We describe System A and System B separately in section 2.1.1 and 2.1.2.

2.1.1 System A

An internal olefin sulfonate, IOS1518, is used as the foaming surfactant in the foam flooding experiments in System A. This product was manufactured by Stepan (19.42 wt% activity, Lot.# 18239-032708) with a trade name of Petrostep S-2A. Silica sand 20/40 (U.S. Silica Company) is used to pack the porous media in the 1-D glass column. The sand pack has a diameter of 2.58 cm and a length of 27.5 cm. A permeability of 158.0 darcy and a porosity of 36.0% are determined for this system.

In System A, the water saturation in porous media is measured by weighing the sand pack using a balance (Sartorius Balance BP 3100S, USA) after all valves are closed and the sand pack is separated from other equipment. Zero water saturation corresponds to the weight of fresh sand pack before water injection; 100% water saturation corresponds to the weight of water-saturated sand pack after water injection to the system which was previously filled with CO₂. By excluding the dead volume (1.5 ml), a linear relationship between the water saturation and the weight of the sand pack is employed to determine the water
saturation at a specific weight. After this calibration, surfactant solutions are used as the aqueous phase in foam experiments other than water. The density difference between the surfactant solution and water is used to calculate the aqueous phase saturation based on the method described above.

The sand pack is filled with DI water before the experiment. IOS1518 with a final concentration of 0.2 wt% is dissolved in brine and the salinity is fixed at 1.0 wt% NaCl. 2 PV of surfactant solution is injected to the porous media to minimize surfactant adsorption. A series of steady-state co-injection experiments start from 99% air injection displacing surfactant solution and end up with 10% air injection. The experimental setup of System A is shown in Figure 1(a). Surfactant solution is co-injected with air to the sand pack using syringe pumps (Harvard Apparatus MODEL 22) at a total superficial velocity \( u_w + u_g \) of 20 ft/day. Because of the lack of internal taps in the glass sand-pack holder, a pressure transducer (Validyne Engineering MODEL DP7) is located at the inflow end, and it is assumed to be measuring pressure drop across the porous media. The outflow end is open to atmosphere. When a steady state is achieved and the pressure drop reaches a plateau value, the pressure drop is averaged over the period of the steady state. The foam apparent viscosity, which is the pressure gradient normalized with respect to the permeability and the total flux of surfactant solution and gas, is calculated through Eqn (1):

\[
\mu_{\text{foam,app}} = \frac{-k \nabla p}{u_w + u_g} \tag{1}
\]
A transient experiment, shown in Figure 17 in this work, is performed using the same sand pack and a mass flow meter (Matheson Model 8200) to control the flow rate of the injected gas phase. After saturated with DI water, the sand pack is placed vertically and 4.0 PV surfactant solution (0.2 wt% IOS1518 in 1.0 wt% NaCl) is injected from bottom to top to displace DI water. Then the sand pack is placed horizontally to allow N₂ injection to the system. The flow rate of N₂ is 2.212 sccm or 20 ft/day in standard condition.
2.1.2 System B

A proprietary surfactant blend ZA (code name Z-R₁-ZFG₁-A-R₂-AFG) is used as the foaming agent in System B. The ZA blend with 1.0 wt% total surfactant

Figure 1. Schematic of the apparatus for foam experiments in (a) System A and (b) System B.
concentration in synthetic seawater is used in all experiments. The synthetic seawater contains 27.0 g/L NaCl, 1.3 g/L CaCl₂, 11.2 g/L MgCl₂ · 6H₂O and 4.8 g/L Na₂SO₄. All aqueous solutions are prepared with DI water (18.2 M Ω·cm), and 30 ppm of Na₂SO₃ as oxygen scavenger.

The foam experiments are done at 94°C in the setup shown in Figure 1(b). Pre-purified gas nitrogen (1, Matheson Tri Gas Nitrogen Compressed) is injected to the oven controlling the flow rate through a gas flow controller (2, Matheson flow controller Model 8270, 0-20 sccm, using a Matheson mass flow transducer 8272-0421). Check valves (3, 5 Swagelok poppet check valve, 5 psig) are located before and after a heat exchanger (4) which is inside the oven where the temperature is maintained at 94°C. The surfactant solution is continuously mixed and filtered (8, Pump inlet filter 0.5 μm) in the suction of a HPLC pump (9, ISCO Model 2350, 0-10 cm³/min), then passes through a check valve (10, Swagelok poppet check valve, 5 psig) before a heat exchanger (11) located inside the oven. After this point the surfactant solution is mixed in a cross (6) where nitrogen is co-injected before being injected to the sand pack. The thermocouple (7, Omega Type K) measures the co-injection temperature. Between the thermocouple and the bottom of the column there is a tap (12) that measures gage inlet pressure using the pressure sensor (24, Validyne Engineering MODEL DP7). The mixture of nitrogen and surfactant solutions are fed into the packed 20-40 mesh silica sand column (ID=2.29 cm, L=38.1 cm, k=100 darcy, 20/40 Ottawa Silica Sand US-Silica). Three internal taps (13, 14 and 15) are located at different heights to measure pressure drops across the packed bed every 6 inch (distances from
bottom to the taps are 1½ in, 7 ½ in and 13 ½ in respectively). The effluent of the
column is passed through a check valve (16, Swagelok poppet check valve, 5
psig) and mixed with a small continuous water flow rate (ca. 0.1 cm³/min), using
an ISCO pump (20, ISCO Model 260D Syringe Pump with ISCO Series Pump
Controller, 0.001 -107 ml/min, max pressure 7500 psi). This pump (20) maintains
a continuous injection of water to keep the relief valve open (18, Swagelok
proportional relief valve RL3 10-225 psi), in order to minimize oscillation at the
outlet pressure, acting as a back pressure regulator. The relief valve (18) is set
up in the range of pressure from 30-40 psig. The setup pressure to open will
depend on the desired conditions of experiment, and to prevent flashing of any
component within the liquid blends. The outlet of the relief valve is conducted to a
heat exchanger (19) to cool the effluent. The cooled effluent is collected in vials
(23). Back-pressure is kept constant by a special arrangement even during more
than one-phase flow, where small amount of water is constantly injected to keep
flow-control open all the time. Liquid-gas ratio is measured when desired by
flipping 3-way valve (21) into an inverted burette (22). The history of the pressure
at different times during the experiments is recorded with a PC using a Validyne
UPC2100 PCI Sensor Interface Card to process the signal of the Validyne
stainless steel low pressure sensors (24,25,26,27 Validyne DP7).

Note that the two heat exchangers have a NTU (the Number of Transfer
Units) higher than 4 at flow conditions so injection of gas and liquid will be at
oven temperature. The heat exchanger to cool down the effluent operates at 0°C.
Steady state is considered when the pressure reading does not change in time,
and when the steady flow condition is reached (i.e. injection flow rates equals production flow rates).

### 2.2 Foam modeling technique

The STARS™ foam model is employed in this work. This foam model assumes local steady state, which means that foam creation and decay mechanisms occur relatively fast compared with flow through porous media. Based on the reported literature, the STARS™ foam model is shown in Eqns (2) and (3)

\[
k_f^f = k_{rg}^f \times FM \quad \text{..........................................................(2)}
\]

\[
FM = \frac{1}{1 + \text{fmmob} \times F_1 \times F_2 \times F_3 \times F_4 \times F_5 \times F_6} \quad \text{..........................................................(3)}
\]

Eqns (2) and (3) indicate that this foam model uses a mobility reduction factor called \(FM\) to change the relative permeability of the gas phase in the presence of foam. The mobility reduction factor \(FM\) includes 6 functions \((F_1 \text{ to } F_6)\), subscripts corresponding to different functions may vary in different versions) to describe different factors on gas mobility reduction. In this work we mainly focus on estimating the parameters in the function \(F_2\) (water-saturation-dependent function) shown in Eqn (4).

\[
F_2 = 0.5 \times \frac{\text{arctan}[\text{edry}(S_w - \text{findry})]}{\pi} \quad \text{..........................................................(4)}
\]
The details of the functions, $F_1$ (surfactant-concentration-dependent function), $F_3$ (oil-saturation-dependent function), $F_4$ (gas-velocity-dependent function), $F_5$ (critical-capillary-number-dependent function) and $F_6$ (salinity-dependent function) can be found elsewhere and are not discussed here.

3. Results and discussion

3.1 Experimental results of foam flooding

3.1.1 Foam experiments in System A

The effect of foam quality (gas fraction) on foam apparent viscosity in System A is shown in Figure 2. We observe two distinctive foam regimes in Figure 2: the high-quality (high-gas-fraction) regime and the low-quality (low-gas-fraction) regime. The foam apparent viscosity increases when injected gas fraction increases in the low-quality regime, and decreases when injected gas fraction increases in the high-quality regime. At the boundary of the two regimes, foam obtains its maximum apparent viscosity at a given surfactant concentration and total superficial velocity. The injected gas fraction at the boundary of these two regimes is known as transition foam quality ($f'_g$).
The effect of foam quality ($f_g$) on the saturation of aqueous phase ($S_w$) is shown in Figure 3. Typical fraction flow curves show that higher $f_g$ (or lower $f_w$) results in lower $S_w$ in the absence of foam. The effect of $f_g$ on $S_w$ in the presence of foam is investigated within the range of $f_g$ from 10% to 99%, and it is found that
$S_w$ is relatively constant over a wide range of foam qualities.

![Saturation of aqueous phase](image)

Figure 3. The effect of foam quality on aqueous saturation in System A. The total superficial velocity is 20 ft/day.

3.1.2 Foam experiments in System B
A set of experiments is conducted at a total superficial velocity of 31 ft/day in System B as shown in Figure 4. The data points show that an increase in injected gas fraction leads to an increase in foam apparent viscosity. This behavior is typical observed in the low-quality foam regime, where bubble trapping and mobilization is the dominating factor for foam strength. It also indicates that the foaming system in Figure 4 is robust, and that the foam may not dry out even at an injected gas fraction of 88%.

3.2 Foam modeling technique with the function $F_2$

3.2.1 Distinction between $S'_w$ and $f_{mdry}$
By assuming incompressible, isothermal flow and the absence of capillary pressure and using 1-D Darcy's law we have

\[ u_w = \frac{-k k_{rw} \nabla p}{\mu_w} \] .................................................................(5)

\[ u_g = \frac{-k f k_{rg} \nabla p}{\mu_g} \] .................................................................(6)

Combined with Eqn (1) we obtain

\[ \mu_{\text{foam,app}} = \frac{k \nabla p}{kk_{rw} \nabla p + kk_{rg} \nabla p} = \frac{1}{k_{rw}(S_w) + k_{rg}(S_g)} \] .................................................................(7)

Note that the compressibility of the gas phase affects flow rates of gas, which leads to an increase in foam quality downstream than upstream in 1-D steady-state experiments. The gas compressibility and the pressure-dependent effective permeability result in a modification to Eqn (6) \(^{45}\), which is not included here.

Meanwhile, the relative permeability is a function of saturation, and in two-phase flow we have

\[ S_w + S_g = 1 \] .................................................................(8)

\[ k_{rw} = k_{rw}^0 \left( \frac{S_w - S_{wc}}{1 - S_p - S_{wc}} \right)^{n_w} \] .................................................................(9)

\[ k_{rg} = k_{rg}^0 \left( \frac{1 - S_p - S_{wc}}{1 - S_p - S_{wc}} \right)^{n_g} \frac{1}{1 + \text{fmmob} \times F_2} \] .................................................................(10)
Here the function $F_2$ is described in Eqn (4). According to Eqns (7)-(10), $\mu_{\text{foam,app}}$ is a function of $S_w$ when other parameters are fixed.

Meanwhile, to calculate gas fractional flow from Eqns (5) and (6), we have

$$f_g = \frac{u_g}{u_w + u_g} = \frac{1}{1 + \frac{k_{rw}(S_w)}{\mu_w} \cdot \frac{\mu_g}{k_{rg}^{f}(S_g)}}$$

Therefore, in a two-phase system $f_g$ is also a function of $S_w$ only if other parameters stay unchanged. Through Eqns (7) and (11), one can calculate $\mu_{\text{foam,app}}$ and $f_g$ as functions of $S_w$.

Consider the example case where $epdry = 500$, $fmmob = 10000$ and $fmdry = 0.1$, $\mu_{\text{foam,app}}$ vs. $S_w$ is shown in Figure 5 using the parameters for System A in Table 1. Note that we use a preset value (500) of $epdry$ to perform the model fit in this work, unless otherwise specified. The choice of $epdry$ and the effect of $epdry$ on foam modeling are discussed in the following section 3.2.4.

We use a connate water saturation ($S_{wc}$) of 0.07 to match experimentally measured water saturation, which is discussed in section 3.2.3.

The parameters in relative permeability curves in Table 1 are obtained from the literature for unconsolidated sandpacks. Foam parameters are sensitive to the parameters in relative permeability curves, especially the exponent in the $k_{rw}$ curve. Note that an exponent close to 4 is usually seen in the relative
permeability curve of the wetting phase, while the exponent 1.96 used here is not typical. We use these parameters in Table 1 to demonstrate how to estimate foam parameters in the dry-out function. The sensitive of foam modeling parameters to the exponents in relative permeability curves will be discussed in a following paper.

Table 1. Parameters for foam modeling in this work

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<td>epdry</td>
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<tr>
<td>$S_{wc}$</td>
<td>0.07</td>
<td></td>
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<tr>
<td>$S_{gr}$</td>
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<tr>
<td>$\mu_w (Pa \cdot s)$ (System A)</td>
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<td>$\mu_w (Pa \cdot s)$ (System B)</td>
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<tr>
<td>$\mu_g (Pa \cdot s)$ (System A)</td>
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<tr>
<td>$\mu_g (Pa \cdot s)$ (System B)</td>
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<td>$k_{rw}^0$</td>
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</tr>
<tr>
<td>$n_g$</td>
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<td>23</td>
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Figure 5. Foam apparent viscosity as a function of water saturation using $f_{nmob} = 10000$ and $f_{mdry} = 0.1$: (a) full profile; (b) close-up profile near $f_{mdry}$. The rest of the parameters used are shown in Table 1 (System A) with a preset $ep_{dry}$ of 500.
There exists a maximum foam apparent viscosity ($\mu_{\text{foam,app}}^{t}$) in Figure 5(a), which corresponds to the transition water saturation ($S_{w}^{t}$) and the transition foam quality ($f_{g}^{t}$). However, $S_{w}^{t}$ is not equivalent to the parameter $f_{\text{ndry}}$, which is designed to be the critical water saturation in the STARS$^{\text{TM}}$ foam model$^{33}$ as indicated in Figure 5(b). In the case of $\text{fmmob}=10000$ and $f_{\text{ndry}}=0.1000$, we obtain $S_{w}^{t} = 0.1071$ through a 1-D unconstrained nonlinear minimization function “fminsearch” in MATLAB$^{49}$.

Despite the small difference between $f_{\text{ndry}}$ and $S_{w}^{t}$ in this example, assuming $S_{w}^{t}$ to be equal to $f_{\text{ndry}}$ can cause significant error in calculating the transition foam quality $f_{g}^{t}$ as shown in Figure 6(b). This is due to the fact that the $f_{g} - S_{w}$ curve is steep near $S_{w} = f_{\text{ndry}}$ in order to describe the effect of the limiting capillary pressure (Figure 6). Therefore, $S_{w}^{t}$ needs to be calculated in the algorithm instead of being assumed to be the parameter $f_{\text{ndry}}$. 
Figure 6. Gas fractional flow as a function of water saturation using $f_{mmob} = 10000$ and $f_{mdry} = 0.1$: (a) full profile; (b) close-up profile near $f_{mdry}$. The rest of the parameters are used as shown in Table 1 (System A) with a preset $ep_{dry}$ of 500.
Note that $g$ is a monotonically decreasing function of $S_w$ according to Eqn (11), which indicates that $\frac{df}{dS_w|_{S_w=S_w^t}} < 0$. Thus, to characterize the transition foam quality $f^t_g$ we have

$$\frac{d\mu_{\text{foam,app}}}{df_g} \bigg|_{f_g=f_g^t} = \frac{\frac{d\mu_{\text{foam,app}}}{dS_w}}{\frac{df}{dS_w}}|_{S_w=S_w^t} = 0 \quad \cdots \quad (12)$$

which is equivalent to

$$\frac{d\mu_{\text{foam,app}}}{dS_w} \bigg|_{S_w=S_w^t} = 0 \quad \cdots \quad (13)$$

Thus, one is able to calculate $f^t_g$ through Eqn (11) using $\frac{d\mu_{\text{foam,app}}}{dS_w} \bigg|_{S_w=S_w^t} = 0$

or $\max_{S_w} \mu_{\text{foam,app}}(S_w) = \mu_{\text{foam,app}}(S_w^t)$ to identify $S_w^t$.

### 3.2.2 Hybrid contour plot method

Cheng and coworkers\(^{33}\) showed how to fit the parameters to steady-state laboratory coreflood data using the STARS\(^{TM}\) foam model. This method is based on the hypothesis that the experimental $\nabla p$ data is vertical in the high-quality regime and horizontal in the low-quality regime in order to draw the contours of $\nabla p$ to fit the model parameters. If the available experimental data is obtained at a
fixed total velocity of both gas and liquid, we propose a method to match a single point with the highest foam apparent viscosity ($\mu_{\text{foam,app}}^i$) and the transition foam quality ($f_g^i$) in this set of experimental data. Additionally, as pointed in section 3.2.1, we program a routine to compute $S_w^i$ instead of using $S_w = f_{\text{dry}}$ to estimate $f_g^i$ in order to improve the accuracy of our calculation.

The procedure tackles three equations with three unknown variables: $S_w^i$, $f_{\text{mob}}$ and $f_{\text{dry}}$:

\[
\mu_{\text{foam,app}}^i(\text{measured}) = \frac{1}{\frac{k_{rw}(S_w^i)}{\mu_w} + \frac{k_{rg}(S_w^i, f_{\text{mob}}, f_{\text{dry}})}{\mu_g}} \tag{14}
\]

\[
f_g^i(\text{measured}) = \frac{1}{1 + \frac{k_{rw}(S_w^i)}{\mu_w} \cdot \frac{\mu_g}{k_{rg}(S_w^i, f_{\text{mob}}, f_{\text{dry}})}} \tag{15}
\]

\[
\max_{S_w} \mu_{\text{foam,app}}(S_w, f_{\text{mob}}, f_{\text{dry}}) = \mu_{\text{foam,app}}(S_w^i, f_{\text{mob}}, f_{\text{dry}}) \tag{16}
\]

Note that the explicit expression of Eqn (16) can be written with the substitution of Eqns (7)-(10). There is more than one way to solve Eqns (14) to (16). For example, $S_w^i$ can be solved from $k_{rw}(S_w^i)$ by eliminating $k_{rg}(S_w^i, f_{\text{mob}}, f_{\text{dry}})$ from Eqns (14) and (15). Then only two variables, $f_{\text{mob}}$ and $f_{\text{dry}}$, are left with two constrains. One can solve $f_{\text{mob}}$ and $f_{\text{dry}}$ using the “fzero” function in MATLAB. However, one should keep in mind that only
the roots which satisfy \( S_{wc} < S_w^f < 1 - S_{gr} , \) \( \text{fmmob} > 0 \) and \( S_{wc} < \text{fmdry} < 1 - S_{gr} \) are acceptable for the estimation of the parameters in the foam model. The problem can also be solved graphically with a hybrid contour plot method shown in Figure 7.

![Flow chart for matching transition foam flow in porous media at steady state using the proposed hybrid contour plot method.](image)

Figure 7. Flow chart for matching transition foam flow in porous media at steady state using the proposed hybrid contour plot method.
We show the matching process of 0.2 wt% IOS1518 in Figure 2 as an example to explain the procedure in Figure 7 in detail. In this case, \( f_{g}^{i} = 0.5 \) and \( \mu_{\text{foam,app}} = 420.5 \text{ cp} \) are obtained from the experimental data in Figure 2.

According to the procedure in Figure 7, contour plots of \( f_{g}^{i} \) and \( \mu_{\text{foam,app}} \) (at \( f_{g} = f_{g,\text{measured}}^{i} \)) with respect to \( f_{\text{mob}} \) and \( f_{\text{dry}} \) using MATLAB are drawn in Figure 8 and Figure 9, respectively.

\[ \text{Contour of transition foam quality} \]

Figure 8. Contour plot of transition foam quality as a function of \( f_{\text{mob}} \) and \( f_{\text{dry}} \). The rest of the parameters are used as shown in Table 1 (System A) with a preset \( e_{\text{p dry}} \) of 500.

Superposition of Figure 8 and 9 leads to a hybrid contour plot shown in Figure 10. In Figure 10, the red curve represents the contour line of \( f_{g}^{i} = 0.5 \) in Figure 8, and the blue curve represents the contour line of \( \mu_{\text{foam,app}} = 420.5 \text{ cp} \) in...
Figure 9. The point where the red curve and the blue curve intersect indicates the
dparameters we search for matching the experimental data (\( f_g^i = 0.5 \) and
\( \mu_{\text{foam,app}}^i = 420.5 \text{ cp} \)). In this case, we obtain \( f_{\text{mob}} = 47094 \) and \( f_{\text{dry}} = 0.1006 \)
from Figure 10 after enlarging the plot in MATLAB.

Figure 9. Contour plot of foam apparent viscosity as a function of \( f_{\text{mob}} \) and \( f_{\text{dry}} \).
Injected gas fraction is set to be 0.5. The rest of the parameters are used as shown in Table 1 (System A) with a preset \( \varepsilon_{\text{dry}} \) of 500.
Using $f_{\text{mmob}} = 47094$ and $f_{\text{mdry}} = 0.1006$, the parameters in Table 1 (System A) and Eqns (7) and (11), we compare the modeling profile of foam apparent viscosity with the experimental data in Figure 11. The simulation results match the experimental data at the transition foam quality, indicating that our calculation is accurate and the proposed procedure in Figure 7 is valid. As shown in Figure 11, both high-quality and low-quality foam regimes are well modeled using this approach with the STARS\textsuperscript{TM} foam model.
Using the same approach, we model the experimental data at a total superficial velocity of 31 ft/day (Figure 4) in System B with the $F_2$ function. The rest of the parameters are listed in Table 1 (System B). Since no transition foam quality is observed in the four blue data points in Figure 4, we assume that the one with the highest apparent viscosity to be the data point at the transition foam quality ($f_{gr}^t = 0.879$ and $\mu_{foam,app}^t = 624.99$ cp). Using the method proposed in section 3.2.2, one can obtain the parameters $f_{mob} = 40725$ and $f_{dry} = 0.0731$ as shown in Figure 12. The modeling result matches the trend of
the experimental data with some underestimation of the data points at low gas fractions.

![Foam apparent viscosity (1 wt% ZA blend)](image)

Figure 12. Comparison between experimental result (Figure 4) and model fit (\( fm_{mob} = 40725 \) and \( fm_{dry} = 0.0731 \)) for the ZA blend system (System B). The rest of the parameters are used as shown in Table 1 (System B) with a preset \( epdry \) of 500.

### 3.2.3 Comparison in water saturation

We compare the water saturation profile for System A in Figure 13 using the parameters in Table 1 and Figure 11. The modeling results are relatively close to the measured water saturation. Note that we use \( S_{wc} = 0.07 \) in Table 1, which enables us to match experimental data more closely at the low-quality foam regime. As initially both \( S_{wc} \) and \( epdry \) are unknown, a good practice may be to
use a value of $S_{wc}$ for a similar system in the literature (0.04, for example\textsuperscript{23}).

Then a search for the optimal $epdry$ can be conducted (see section 3.2.4) and other parameters such as $fnumob$ and $fmdry$ can be estimated as well. After that, one can compare the water saturation between modeling results and experimental data. If the water saturation is overestimated or underestimated, some fine tunings of $S_{wc}$ are needed to obtain good match to experimental data. In this case a slight increase in $S_{wc}$ from 0.04 to 0.07 results in an increase in estimated $fmdry$, but no significant change is found in optimized $epdry$ value (around 500) for both transient and steady-state experiments.
3.2.4 Effect of epdry

The epdry parameter regulates the slope of the $F_2$ curve near $f_{mdry}$. It was found that reducing $epdry$ causes a deviation of $\nabla p$ contours from vertical lines in the high-quality regime$^{33}$. Here we show how changes in $epdry$ affects the model fit to experimental data. Similar to Figure 5 but with various values of $epdry$, we plot foam apparent viscosity as a function of water saturation in Figure 14 using $f_{nmob} = 10000$, $f_{mdry} = 0.1000$. 

![Saturation profile of aqueous phase (0.2 wt% IOS1518)](image)
Figure 14. Foam apparent viscosity as a function of water saturation using $f_{nmob} = 10000$ and $f_{mdry} = 0.1$: (a) full profile; (b) close-up profile near $f_{mdry}$. Except for $e_{dry}$, the rest of the parameters are used as shown in Table 1 (System A).
Figure 14 shows the effect of $epdry$ on the $\mu_{\text{foam,app}} - S'_w$ profile provided that $fmmob$ and $fmdry$ stay unchanged. It is found that as $epdry$ increases the maximum apparent viscosity increases until it reaches a plateau value (179 cp in this case). Another observation is that the difference between $fmdry$ and $S'_w$ is more significant as $epdry$ decreases, indicating a more gradual transition between the high-quality and low-quality foam regimes.

Figure 15. Model fit to experimental data with different preset $epdry$. $fmmob$ and $fmdry$ are calculated using the method shown in Figure 7 for each value of $epdry$. Except for $epdry$, the rest of the parameters are used as shown in Table 1 (System A).
To identify which value of $\text{epdry}$ provides best fit to the steady-state experiments, we define the residual sum of squares (RSS) by comparing the relative difference between modeling results and experimental data:

$$
\text{RSS} = \sum_{i=1}^{n} \left( \frac{\mu_i, \text{calculated} - \mu_i, \text{measured}}{\mu_i, \text{measured}} \right)^2
$$

(17)

The calculated foam apparent viscosities are based on the hybrid contour plot method shown in Figures 7 to 10. We use the 11 experimental data points in the 0.2 wt% IOS 1518 system as an example to show how RSS varies with $\text{epdry}$. The model fit with different preset $\text{epdry}$ is shown in Figure 15. As indicated in Figure 16. Residual sum of squares as a function of $\text{epdry}$ for the case with 0.2 wt% IOS 1518 steady-state experiments. $\text{fmmob}$ and $\text{fmdry}$ are calculated using the method shown in Figure 7 for each value of $\text{epdry}$. Except for $\text{epdry}$, the rest of the parameters are used as shown in Table 1 (System A).
Figure 16, there exists a minimal RSS in a range between 100 and 1,000,000 for \( epdry \). For this set of experimental data, the minimum of RSS is obtained with an \( epdry \) value of around 500. Nevertheless, for other values of \( epdry \) which are larger than 500, the fit to steady-state data also seems reasonable (Figure 15) with relatively small RSS values shown in Figure 16.

An additional transient experiment may further narrow down the range of \( epdry \) which we should consider. The experimental procedure is described in section 2.1.1, which is basically gas injection to a surfactant-solution-saturated sand pack at a constant flow rate. We program an in-house foam simulator using the IMPES (implicit in pressure and explicit in saturation) finite difference algorithm to conduct transient simulation and to match experimental data. 500 grid blocks (NX=500) are used for the transient simulation and the rest of the parameters are consistent with those in Table 1 except for \( epdry \). The capillary pressure function is disabled and the algorithm assumes 1-D incompressible isothermal flow.
As shown in Figure 17, foam apparent viscosity gradually goes up when gas is injected into the porous medium. The maximal apparent viscosity (39.6 cp) is achieved at 1.34 TPV in the experiment. After 1.34 TPV, a typical dry-out effect is observed and foam gradually loses the strength as an effect of the limiting capillary pressure. Among different sets of model fit using the estimation method proposed in Figure 7, it appears that the parameter set with an $epdry$ slightly smaller than 500 is close to experimental observation especially after gas breakthrough. Figure 17 also reveals an issue with the STARS foam model: foam generation is faster in the simulation than the experiment. A population balance
model \(^{12}\) may describe in-situ foam generation and coalescence more accurately than the local steady-state STARS foam model.

Nevertheless, the above results clearly indicate that an \(e_{p_{\text{dry}}r}\) value around 500 is suitable for matching both steady-state and transient experiments in the case of 0.2 wt\% IOS 1518 (System A). However, how the value of \(e_{p_{\text{dry}}r}\) depends on other properties in the system, such as surfactant type and concentration, still needs further investigation.

4. Conclusions

In summary, we propose a facile approach to obtain the parameters in the STARS\(^{\text{TM}}\) foam model for foam simulation at steady state. Two systems with different experimental conditions are successfully modeled using the proposed technique. To achieve a better accuracy of model fit at the transition foam quality, the difference between the foam model parameter \(f_{m_{\text{dry}}}\) and the transition water saturation \(S_{w}^{t}\) is identified and a method to precisely calculate \(S_{w}^{t}\) is developed. It is shown that the difference between \(f_{m_{\text{dry}}}\) and \(S_{w}^{t}\) becomes large when \(e_{p_{\text{dry}}r}\) becomes small, which corresponds to a more gradual transition between the high-quality foam regime and the low-quality foam regime. We show how a different preset \(e_{p_{\text{dry}}r}\) leads to different \(f_{m_{\text{mob}}}\) and \(f_{m_{\text{dry}}}\) for fitting a set of experimental data using the proposed approach. For the 0.2 wt\% IOS1518 system, we demonstrate that an additional transient experiment is necessary to
narrow down the range for optimized $epdry$ scan. The combination of steady-state and transient experiments may lead to an estimation of a unique set of parameters in the dry-out foam model. The effect of surfactant concentration and fluid velocity on parameter estimation in the foam model will be discussed in a following paper.

5. Nomenclature

$Ca$ = capillary number

$epdry$ = a parameter regulating the slope of $F_{2}$ curve near $fmdry$

$f$ = fractional flow

$f_{g}^{t}$ = transition foam quality where the maximum foam apparent viscosity is achieved

$FM$ = a dimensionless interpolation factor in STARS$^{TM}$ foam model

$fmdry$ = critical water saturation in STARS$^{TM}$ foam model

$fnmob$ = reference mobility reduction factor in STARS$^{TM}$ foam model

$F_{2}$ = a function describing the contribution of aqueous phase saturation in the STARS$^{TM}$ foam model

$k$ = permeability, $m^{2}$

$k_{r}$ = relative permeability

$k_{rw}^{0} = \text{end-point relative permeability of aqueous phase}$

$k_{rg}^{0} = \text{end-point relative permeability of gaseous phase}$

$p$ = pressure, $Pa$
\( P_c \) = capillary pressure, \( Pa \)

\( P_c^* \) = limiting capillary pressure, \( Pa \)

\( u \) = superficial (Darcy) velocity, \( m/s \)

\( S \) = saturation

\( S_w^t \) = transition water saturation where the maximum foam apparent viscosity is achieved

\( \mu \) = viscosity, \( Pa \cdot s \)

\( \mu_{\text{foam,app}} \) = foam apparent viscosity, \( Pa \cdot s \)

\( \mu_{\text{foam,app}}^t \) = maximum foam apparent viscosity obtained at the transition foam quality, \( Pa \cdot s \)

\( \sigma \) = interfacial tension, \( N/m \)

Superscripts

\( nf \) = without foam

\( f \) = with foam

\( n_g \) = exponent in \( k_g \) curve

\( n_w \) = exponent in \( k_w \) curve

\( t \) = transition between high-quality and low-quality foam

Subscripts

\( g \) = gaseous phase
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7. References


