ELEC 501 Fall-02
Topics for the Term Project

1. **A survey of balancing related model reduction methods:** A literature survey and comparison of the model reduction algorithms based on balancing certain system gramians. The project includes

   (a) Lyapunov Balancing (the one we discussed in the class)

   (b) Positive Real Balancing (balancing of the passive systems)

   (c) Bounded Real Balancing (balancing of the bounded real systems)

   (d) Stochastic Balancing

   (e) LQG Balancing

   References:

   [1] S. Gugercin and A.C. Antoulas, *On balancing related model reduction methods and the corresponding error bounds*, Technical Report, Rice University, ECE Dept., September 2002.
   [2] R. Ober, *Balanced parameterization of classes of linear systems*, SIAM J. Cont. App., 1991.

2. **Frequency Weighted Balanced Reduction:** The frequency weighted balancing method is used to minimize the error between the full-order model $G(s)$ and the approximant $G_r(s)$ over a specified frequency region, i.e. for some input weighting $W_i(s)$ and the output weighting $W_o(s)$, the goal is to minimize the weighted error

   $$\| W_o(s)(G(s) - G_r(s))W_i(s) \|_\infty .$$

   The project includes a survey of the several weighted balanced reduction algorithms and their applications to numerical problems.

   (a) Enns' weighted balancing method

   (b) Lin and Chiu's weighted balancing method

   (c) Weighted balancing approach of Sreeram *et al.*

   (d) Zhou's self-weighted balancing method

   (e) Gawronski and and Juang's band limited approach

   (f) Gugercin and Antoulas's modification to (e) above.

   References:

   [1] K. Zhou, *Frequency-weighted $\mathcal{L}_\infty$ norm and optimal Hankel norm model reduction*, IEEE Trans. Automat. Contr., vol. 40, No. 10, pp. 1687-1699, 1995.
   [2] S. Gugercin and A.C. Antoulas, *On balancing related model reduction methods and the corresponding error bounds*, Technical Report, Rice University, ECE Dept., September 2002.
   [3] G. Wang, V. Sreeram and W.Q. Liu, *A new frequency weighted balanced truncation method and an error bound*, IEEE Trans. Automat. Contr., Vol. 44, No. 9, pp. 1734-1737, 1999.

3. **Approximate low-rank solutions to large-scale Lyapunov equations:** For large-scale settings, obtaining an exact solution to the Lyapunov equation $A\mathcal{P}+\mathcal{P}A^T+BB^T = 0$, becomes a formidable task. This project studies the approximate solutions, especially the low-rank ones, of the Lyapunov equation; i.e., given A and B, find a low-rank square-root factor $U \in \mathbb{R}^{n\times r}$ with $n \ll r$ so that $\hat{\mathcal{P}} = UU^T$ is an approximate solution to $A\mathcal{P} + \mathcal{P}A^T + BB^T = 0$. The methods which are based on the so-called *Alternating Direction Implicit*

(ADI) iteration are of particular interest.

References:
[1] T. Penzl, *A cyclic low-rank Smith method for large sparse Lyapunov equations*, SIAM J. Sci. Comput., Vol. 21, No. 4, pp: 1401-1418, 2000.
[2] S. Gugercin, D.C. Sorensen, and A.C. Antoulas, *A modified low-rank Smith method for large-scale Lyapunov equations*, to appear in Numerical Analysis, September 2002.
[3] R. A. Smith, *Matrix Equation*, $XA + BX = C$, SIAM J. Appl. Math, 16: 198-201, 1968.

4. **Model reduction of the flexible structures:** The dynamical systems of the form

$$M\ddot{x}(t) + G\dot{x}(t) + Kx(t) = Hu(t), \qquad y(t) = Px(t) + Q\dot{x}(t). \tag{1}$$

where $M\,, G\,, K \in \mathbb{R}^{n\times n}$, $H \in \mathbb{R}^{n\times m}$ and P, $Q \in \mathbb{R}^{p\times n}$ are called the flexible structures (sometimes also called the second order systems). Many mechanical structures are often of this form. The project studies the approximation methods for the model reduction of the flexible structures including

   (a) Krylov-based methods and balanced reduction based on the following equivalent first-order state-space form of the flexible structure:

$$\left.\begin{array}{rcl} \dot{q}(t) & = & \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}G \end{bmatrix} q(t) + \begin{bmatrix} 0 \\ M^{-1}H \end{bmatrix} u(t), \\ y(t) & = & \begin{bmatrix} P & Q \end{bmatrix} q(t), \qquad \text{where } q(t) = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}. \end{array}\right\} \tag{2}$$

   Note that (2) is in the state-space form with the state $q(t)$.

   (b) Balanced reduction directly on the flexible (second-order) form (1).

The main issue here is to find a reduced order model which is directly in the second-order form (1) above or can be transformed back into that form.

References:
[1] D.G. Meyer and S. Srinivasan *Balancing and model reduction for second order form linear systems*, IEEE Trans. Automat. Contr., **41**: 1632-1644, 19996.
[2] K. Meerbergen and F. Tisseur, *The quadratic eigenvalue problem*, SIAM Review, **43**: 235-286, 2001.

5. **Model reduction of the linear periodic time-varying (LPTV) systems:** The discrete-time l-periodic system is defined as

$$x_{k+1} = A_k x_k + B_k u_k, \qquad y_k \;=\; C_k x_k$$

where $A_{i+lj} = A_i \in \mathbb{R}^{n\times n}$, $B_{i+lj} = B_i \in \mathbb{R}^{n\times p}$, and $C_{i+lj} = C_i \in \mathbb{R}^{p\times n}$ for $j \in \mathbb{Z}$, and $l \in \mathbb{Z}_+$; that is, the state-space matrices are periodically time-varying. This projects involves the realization problem and/or the model reduction methods for LPTV systems.

References:
[1] A. Varga, *Periodic Lyapunov equations: some applications and new algorithms*, Int. J. Control, 67: 69-87, 1997.

[2] A. Varga, *Balanced truncation model reduction of periodic systems*, In the Proceedings of the 39^{th} CDC, Sydney, Australia, December, 2000.

6. **Model reduction of the positive real (passive) systems:** This project involves the analysis of the several model reduction methods for the passive systems and their comparison through some numerical examples (for example some RLC circuits).

   (a) Positive real (passive) balanced model reduction

   (b) Gugercin and Antoulas' modified positive real balanced reduction

   (c) Freund's passive model reduction via the Lanczos procedure

   (d) Antoulas' passive model reduction based on the rational Krylov method

   References:
   [1] S. Gugercin and A.C. Antoulas, *On balancing related model reduction methods and the corresponding error bounds*,
   [2] A.C. Antoulas, *Model reduction with stability and passivity constraints*, Tech. Report, ECE Rice University, November 2002.
   [3] Z. Bai, P. Feldman and R. Freund, *Stable and passive reduced-order models based on partial Pade approximation via the Lanczos procedure*, Bell Lab., Lucent Technologies, Numerical Analysis Manuscript 97/3-10, November 1997.
   [4] R. Freund, *Passive reduced-order models for interconnect simulation and their computation via Krylov subspace algorithms*, Proceedings ACM DAC 99, New Orleans, 1999.
   [5] P. Feldman and R.W. Freund, *Efficient linear circuit analysis by Pade approximation via the Lanczos process*, IEEE Trans. Computer-Aided Design, vol 14, pp. 639-649.

7. **Rational Krylov method for model reduction of large-scale systems:** In the class, we have discussed the moment matching problem around infinity. However, in many cases one is interested in matching the moments at various selected frequencies to obtain a better approximation over a broad frequency range. The solution in this case is given by the Rational Krylov method of Ruhe. This projects involves the analysis of the Rational Krylov method and its comparison with other reduction techniques.

   References:
   [1] E.J. Grimme, *Krylov Projection Methods for Model Reduction*, Ph.D. Thesis, ECE Dept., U. of Illinois, Urbana-Champaign,(1997).
   [2] A. C. Antoulas, D. C. Sorensen, and S. Gugercin, *A survey of model reduction methods for large scale systems*, Contemporary Mathematics, AMS Publications, 280: 193-219, 2001.
   [3] A.C. Antoulas, *Lectures on the approximation of linear dynamical systems*, Draft, to appear, SIAM Press 2002.
   [4] K. Gallivan, P. Van Dooren, and E. Grimme, *On some recent developments in projection-based model reduction*, in ENUMATH 97 (Heidelberg), World Sci. Publishing, River Edge, NJ, 1998, pp. 98–113, 1998.