ELEC 501 **Serkan Gugercin**
Approximation Theory November 22, 2002

Homework 4

Due Date: **December 6, 2002**

Problem [1]. Eigenvalue Computation using Lanczos and Arnoldi:

Part-a: Non-Symmetric Case: Construct a 10×10 random matrix A using the command `A = randn(10)`.

(i) Apply Arnoldi method to compute the eigenvalues of A for $k = 1 : 10$. At each k, draw the eigenvalues of A and the resultant $k \times k$ Hessenberg matrix on the same plot. For different k values, use different plots so that you can see which eigenvalues are caught at each iteration.

(ii) Repeat (i) above for the two-sided Lanczos process. In this case resultant $k \times k$ matrix would be tridiagonal instead.

Part-b: Symmetric Case Construct a 10×10 randomly generated symmetric matrix A. This can be done as follows: `A = randn(10); A = A+A'`. Then apply the Lanczos to compute the eigenvalues of A for $k = 1 : 10$. As above at each k, draw the eigenvalues of A and the resultant $k \times k$ tridiagonal matrix.

Problem [2]. (Sorensen) Consider the k-step Arnoldi factorization

$$AV_k = V_k H_k + f_k e_k^*$$

with starting vector $v_1 = V_k(:, 1)$. Show that

$$\phi(A)v_1 = V_k \phi(H_k) e_1$$

where $\phi(\xi)$ is a polynomial in ξ of degree less than k. Show also that if the degree of ϕ is k, then

$$\phi(A)v_1 = V_k \phi(H_k) e_1 + \alpha f_k$$

where α is the product of the subdiagonal entries of H_k, i.e. $\alpha = h_{21} h_{32} \cdots h_{k,k-1}$.

Problem [3]. Consider the 10^{th} order Butterworth filter constructed in MATLAB as follows:

$$[\mathtt{A}, \mathtt{B}, \mathtt{C}, \mathtt{D}] = \mathtt{butter}(10, 1, '\mathtt{s}');$$

Try to obtain a 4^{th} order reduced model using **(i)** Arnoldi and **(ii)** Two-sided Lanczos procedures. Do the algorithms work without breakdown? If any of two does, what is the transfer function of the reduced order model? Now, to avoid the difficulties encountered, apply a state-space transformation to the quadruple (A, B, C, D). Are the problems overcome? Explain your reasons.

Problem [4]. Consider a circuit consisting of 10 sections interconnected in cascade; each section is as shown in the Figure 1. The input to this is voltage V applied to the first section; the output is the current I of the first section, as shown.

- Derive a state space representation of this circuit. Plot its frequency response.
- Using the square-root method, compute the reachability and observability gramians and hence find the Hankel singular values. Based on the Hankel singular values, comment on the hardness of the approximating the model. Then approximate the system by means of 10 states making use of the following four approximation methods: Square-root Balanced Truncation Method, POD Method, Arnoldi procedure and Lanczos Procedure. Are the reduced systems stable? Plot the frequency response of the reduced systems and the full order model on the same graph. Compute the error systems and $\mathcal{H}_\infty$ error norms. Also draw the frequency response of the error systems on the same plot.

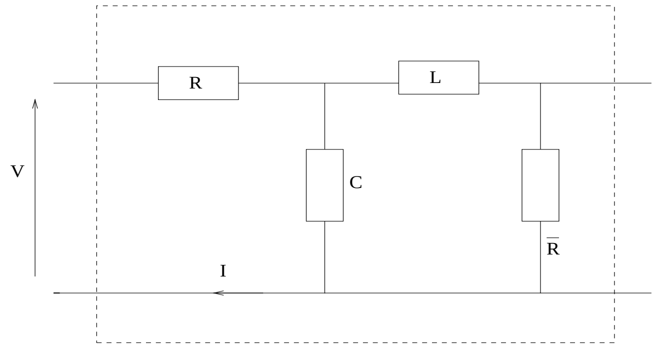

Figure 1: The values of elements are: $R = 0.1\Omega$, $C = 1pF$, $\bar{R} = 100\Omega$, $L = 10pH$.

Problem [5]. Consider the 25^{th} order Elliptic continuous time filter constructed in MATLAB as follows:

$$[\mathtt{A},\mathtt{B},\mathtt{C},\mathtt{D}] = \mathtt{ellip}(25, 1, '\mathtt{s}');$$

Compute the Hankel singular values using the method of Problem [4], and comment on the hardness of the approximating this system? Using the same four methods in Problem [4], reduce the order to k=15. Are the reduced systems stable? Plot the sigma plot of the reduced systems and the full order model on the same graph. Compute the error systems and $\mathcal{H}_\infty$ error norms. Also draw the frequency response of the error systems on the same plot.

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