First Exam

1. The model of a closed system consists of six resources where jobs receive service. Four of the resources are queueing centers, where a single server provides service and jobs must wait while another job is receiving its service. Two of the centers are delay centers; jobs at these centers are only delayed by their own service demands, and never have to wait while the server is busy with some other job.

   The total demands for a job (not per-visit demands) at the four queueing centers are 10, 50, 25, and 15 ms. The total demands of each job (again, not per-visit demand) at the two delay centers are 100 and 300 ms. Note that this model is NOT a central server model.

   (a) At how many jobs does the system saturate? (10 pts)

   (b) Suppose you decided to approximate the throughput curve (a plot of system throughput vs. number of jobs) by a curve that lies exactly halfway between the upper and lower bounds for throughput. What is the maximum possible relative error, both positive and negative, for this approximation and the data given for this system, and at how many jobs does this occur? If $\text{app}(N)$, $\text{up}(N)$, and $\text{low}(N)$ are the approximation, upper bound, and lower bound, respectively, for N jobs in the system, the positive relative error is $\left(\text{up}(N)/\text{app}(N)\right)-1$ and the negative relative error is $\left(\text{low}(N)/\text{app}(N)\right)-1$. (15 pts)

   (c) What can be done to the system in order to double the number of jobs at saturation? Your options are to speed up (or slow down) one or more of the queueing centers. Speeding up a queueing center increases its cost; slowing one down decreases its cost. Discuss the ramifications of each solution that you propose. (10 pts)

2. Suppose you want to use a truncated exponential distribution over the range (0,1) in a stochastic simulation. A truncated exponential distribution over this range has the form $F_X(x) = k\left(1-e^{-\lambda x}\right)$ for $0 \le x \le 1$ and $F_X(x) = 0$ otherwise. Describe at least two algorithms for generating random numbers from this distribution. Be precise. You may assume that you have a $U(0,1)$ random number generator to use. Compare your algorithms on the basis of efficiency, i.e., on the amount of computation required to generate a single random number. (30 pts)

3. (a) A performance analyst simulated a computer system a total of 10 times, each simulation run independent of all the others. She calculated and recorded the sample means for system response time from each of the 10 runs, coming up with the following data:

   2,5,17,3,9,6,4,25,8,1

What is the confidence interval with a 95% confidence level for the mean response time? (10 pts)

(b) The performance analyst was concerned that the width of the confidence interval was too large, and consulted one of her colleagues (who never took Elec 428). He assured her that the width of the confidence interval was large because of the two large "outliers," the values 17 and 25, and that she could reduce the width of the confidence interval simply by discarding these values. She tried this, and discovered that the width of the interval did indeed go down substantially.

Is this a valid method? Why or why not? Be specific. (5 pts)

(c) Another colleague suggested that she try "smoothing" the data, by replacing each value with the average of itself and the preceding and succeeding values, and discarding the first and last values since the first has no predecessor and the last has no successor. She tried this, and was delighted to find that it did make the interval significantly smaller.

Is this a valid method? Why or why not? Be specific. (5 pts)

(d) Yet another colleague (she apparently has many) advised her to take each non-overlapping pair of successive data points (sample means) and replace them by a single value which was their average. This would also "smooth" the data, as it reduced the number of data points to five. What did she determine that this did to the confidence interval width, for the same confidence level? Is this a valid procedure? Why or why not? (5 pts)

(e) In general, what effect would you expect this procedure to have on the width of the interval? (10 pts)