Elec 428
Spring Semester 2002

Final Exam

This is a two-hour, open-book, open-notes, take-home exam. Work the exam on standard 8.5" x 11" paper, **one side only**. If a problem statement appears ambiguous or incomplete, make any *reasonable* assumptions that you feel are necessary to solve the problem and state those assumptions as part of your solution. Return the exam with your solution sheets. The exam is due in DH 1016 by Thursday, May 2, at 12 noon if you are a degree candidate, or by 5 PM on Wednesday, May 8, if you are not. You should plan to have access to matlab or an equivalent numerical analysis tool for linear algebra.

SIGN THE PLEDGE

First Exam

1. A shared memory multiprocessor system consists of two processors and four memory modules. The system has the following characteristics:

   - Each processor chooses the memory module to which it will direct a new request randomly and with equal probability for all modules.
   - A memory module can service only one request at a time. If more than one request is queued at a module at the beginning of a memory cycle, one of the requests is selected at random and serviced. All other requests in the queue at the start of the memory cycle remain in the queue.
   - The memory modules are synchronized; all modules busy concurrently start and complete their memory cycles together.
   - The time for a memory cycle is fixed.
   - Each processor always has <u>two</u> requests outstanding. Consequently, during a memory cycle, a processor may have zero, one, or two requests serviced. If it has one or two requests serviced, it immediately generates one or two new memory service requests, which takes zero time, for the start of the next memory cycle.

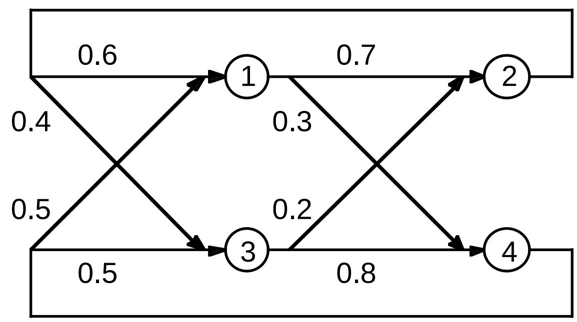
   (a) Define a set of states for a discrete time Markov chain modeling this system that is sufficient to allow you to calculate the average number of memory modules busy during a memory cycle. Use as few states as possible. Be precise. (5 pts)

   (b) Completely describe the Markov chain model by giving all single step transition probabilities. Include both a single step transition probability matrix and a state transition graph in your answer. Draw and label the graph completely and legibly. (20 pts)

   (c) Determine the steady state probabilities for the Markov chain. (5 pts)

   (d) What is the average number of modules busy during a memory cycle? (5 pts)

2. The following diagram represents a small computer network with four switching nodes. The label on the path from node i to node j is the probability that a message leaving switching node i is routed to switching node j.



What are the visit ratios for nodes 2, 3, and 4, if the visit ratio for node 1 is 1? (15 pts)

3. An interactive computer system consists of a cpu and two disks, d1 and d2. The system has a total of 3 interactive users, with one job associated with each user. The cpu and disks each use a FCFS queueing discipline. A job completing service at the cpu goes to d1 with probability 0.65 and d2 with probability 0.3, or requires some action by the user with probability 0.05. A job completing service at d1 or d2 always goes back to the cpu for more service.

   Each time a job visits the cpu, d1, and d2, it requires 10 ms, 30 ms, and 50 ms, respectively, of service on average. On average, each user action requires 100 ms (clearly, the user has been drinking too much caffeine). For some reason, you decide that the service time distributions at the cpu, d1, and d2 are exponential.

   When the user completes his action, his job will attempt to get service from the cpu. However, the computer has only enough memory for two jobs at a time. If a job is given a memory partition, it holds that partition until it prompts the user for further action, at which point the job releases its partition. A job that has been reactivated by the completion of the user action will first attempt to get a memory partition. If no partition is available, the job must wait until another job releases its partition, which is then given to the waiting job.

   (a) What is the approximate system throughput? (20 pts)

   (b) Approximately how many users are active (i.e, approximately how many jobs are NOT at the cpu or either disk) at a time? (5 pts)

4. A shared memory multiprocessor has four processors and four memory modules. Its performance characteristics are exactly like those of the SMMP model first discussed in class. However, the number of jobs in the system is not fixed. Jobs arrive from the outside world according to an exponential interarrival time distribution and depart from the SMMP when they complete their service requirements.

When one job is in the system, its average service time is exactly equal to the average interarrival time. When there are two jobs in the system, each job is assigned to a different processor, and the two jobs are executed concurrently. However, because of the contention for memory, the combined service rate is not twice the service rate of a single job in the system. Rather, the combined service rate is equal to the single job service rate times the steady state average number of busy memory modules when the SMMP has two jobs, and hence two processors generating memory requests. Similarly, the combined service rate with three and four jobs in the SMMP is equal to the single job service rate times the steady state average number of busy memory modules when the SMMP has three and four jobs, respectively. If the SMMP has more than four jobs, only four of them are being executed, and the other jobs are placed on a queue, waiting for a processor to become free when a job completes its service.

Assume that the job service times are exponentially distributed with the appropriate mean service times as discussed above.

(a) What is the average number of jobs in the system? (15 pts)

(b) What is the average time a job spends in the system, including its waiting time? (5 pts)

(c) What is the probability that the SMMP is completely idle (no jobs in the system)? (5 pts)