Elec 428 - Spring 2006
Homework 5

All homework assignments and projects in this course are covered by the Honor Code. You may work on the homework problems together but all written work must be exclusively your own (do not look at anyone else's written work or at solutions from previous years). Late homework assignments lose credit at the rate of 10% per day.

Due date: Wednesday, April 19

1. A computer is used to process images arriving from five independent sources. Each source sends an image to the computer at random intervals exponentially distributed in length. One of the sources sends images at the rate of 10 images per second. Each of the other four sources sends images at half that rate.

   The computer processes images in three phases. In the first phase, it scans the entire image pixel by pixel, taking a constant 0.005 seconds per image. The second phase compresses the image, and takes an amount of time that depends on the image complexity. The compression time is a random variable with a uniform distribution between 0.01 and 0.02 seconds. The third and final phase, which encodes the compressed image, takes either 0.02 seconds with probability 0.2 or 0.01 seconds with probability 0.8. The phases are independent of one another. Only one image can be processed at a time, and an image must go through all three phases before processing of the next image can begin. The computer is able to buffer as many images as necessary; images are never lost.

   What is the average number of images either being processed or waiting to be processed?

2. (a) A switch in a computer communications network has eight input links and eight output links. Messages arrive over each input link at rate λ messages per second (a Poisson process). Under normal circumstances, each message that arrives at the switch is routed to output links 1-7 with probability 0.1 each and to output link 8 with probability 0.3. A link can transmit only one message at a time. The time to transmit a message over an output link is exponentially distributed with mean $4/(5\lambda)$.

   What is the average number of messages in each output link message queue (including any messages that are in the process of being transmitted on the link)?

   (b) To relieve the congestion on link 8, the switch employs a congestion control algorithm. If the number of messages in the queue of output link 8 (including the message being transmitted) is four, a message that normally would be routed to output link 8 is routed to one of the other seven output links, with equal probability for each link. What is the average number of messages in output link 8's queue?

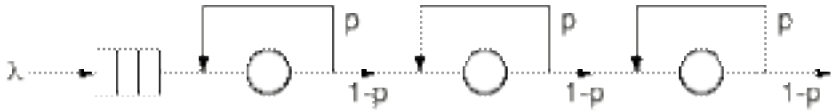
   (c) What is the approximate average number of messages in each of the other output link queues when the congestion control algorithm of part (b) is used?

3. A data concentrator has eight input lines and a single output line. Messages arrive on each of the input lines with exponentially distributed inter-arrival times, and message arrivals times are independent of one another. The arrival rate over each input line is 10 messages per second. Each message that arrives must be processed by the data

concentrator and then transmitted over the output line. Processing a message takes 5 μsec per byte. Message lengths are uniformly distributed between 500 and 1500 bytes. The output link speed is 100 megabits per second. Processing and transmitting a message are not overlapped, and the concentrator can deal with only one message at a time (i.e., the concentrator cannot start on a new message until the previous message has been both processed and transmitted).

What is the average number of messages at the data concentrator?

4. A system is modeled by the following single queue.



The single server consists of 3 stages that are identical. The one job in service visits each stage in turn. However, each stage may be repeated an arbitrary number of times. The probability that a job that has just received service at a stage repeats the stage is p; the probability that it goes on to the next stage is $1-p$. Each time a job receives service at a stage, its fixed service time is 1. Hence, a job that visits the first stage just once receives a total service of 1 from the first stage; if it visits the first stage twice (is fed back once), it receives a total service of 2 from the first stage; etc.

If $\lambda = 1/10$ (Poisson arrival process) and $p = 1/3$, what is the average response time (total of service time and waiting time) for the queue?