Elec 428 - Spring 2006
Homework 6

All homework assignments and projects in this course are covered by the Honor Code. You may work on the homework problems together but all written work must be exclusively your own (do not look at anyone else's written work or at solutions from previous years). Late homework assignments lose credit at the rate of 10% per day.

Due date: Wednesday, April 5, 2006

1. An unnamed university recently decided to fix its parking problems. The consultant hired to develop a parking plan (whose previous parking scheme, locally referred to as the Southwest Freeway, was a smashing success) came up with a novel idea. Since parking places were needed only because people drove cars to the university, and since people would not drive cars to the university if there were no parking places, the parking problem could be fixed by eliminating all parking places. The university's Board of Governors was understandably impressed by the idea, but when the president of the university pointed out that this might not be entirely practical, the board asked the consultant to revise her plan slightly. She quickly developed a new scheme whereby the university would have a total of two parking places. Realizing that there might be more than two people wanting to park in these spaces, she planned to sell more than two parking permits, but to charge $100,000 per permit to keep down the number of people applying for permits.

   Much to her surprise, the board accepted the proposal and instituted the plan. A total of four people (all from the business school) applied for and were given parking stickers. As a follow-up, the consultant studied the parking on campus for the next month and presented the following observations at the next board meeting.

   (i) Cars arrived on campus to park according to a Poisson process, with rate 0.2 arrivals/hour for each car not currently on campus (either parked or waiting to park). Arrivals were completely independent.

   (ii) Drivers who succeeded in finding a vacant parking space parked for an exponentially distributed amount of time with mean 2 hours, independently of the parking time of any other car.

   (iii) Drivers who arrived on campus but failed to find an open parking space vultured (i.e., waited in the middle of the road with their engines running) until a parked car left and then immediately took the vacated parking space. If more than one car was vulturing when a space opened up, the space was filled on a first-come-first-served basis.

   Assume steady-state when answering the following questions.

   (a) What is the probability that both parking places are full?

   (b) What is the probability that a car arrives when both parking places are full?

   (c) What is the average amount of time spent waiting by a vulturing car, given that it does in fact have to vulture?

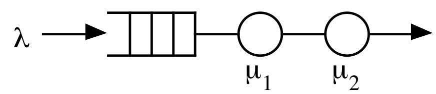
2. A telephone system consisting of two lines services three callers. Each caller makes calls that are exponentially distributed in length, with mean $1/\mu$. A caller whose previous

attempt to make a call was successful (the caller was not blocked) has an exponentially distributed time before attempting the next call, with rate λ. A caller whose previous call attempt was blocked is impatient and tries to call again at twice that rate (2λ), also according to an exponential distribution. The callers make their calls independent of one another. The objective is to determine the average number of calls in progress.

Draw the state transition diagram for this system. Define all states carefully, and clearly label the nodes and arcs of the diagram. Also give the rate generator matrix for this system, being sure to relate the rows (and columns) to their associated states.

3. A computer center has three computers and two repairmen. A working computer has an exponentially distributed time to failure, with rate λ. When a computer fails, only one repairman can work on it at a time. The time to fix a failed computer, once a repairman starts working on it, is exponentially distributed with rate μ. What is the probability that a computer which fails will not be serviced immediately if $\lambda = \mu$? What is the fraction of time that a computer will be broken but not under repair if $\lambda = \mu$?

4. Consider the following queueing model.



Jobs arrive according to a Poisson process with rate $\lambda = 1$ and join the FCFS queue. When a job reaches the head of the queue, it receives service from two servers in a row. Each server has an exponentially distributed service time distribution, with mean service times $1/\mu_1 = 1/2$ for the first server and $1/\mu_2 = 1/4$ for the second server. A job must complete service at <u>both</u> servers before another job is allowed to begin service at the first server. The service times are independent random variables.

Draw the state transition rate diagram for the above system, assuming a maximum queue length (including the job in service) of 3. Be sure to explain the diagram fully. What is the probability that both servers are idle?

5. A system consists of a single server. Jobs arrive at the server according to a Poisson process. The system permits no more than 4 jobs to be at the server, including any job currently in service. Jobs that arrive when the server is full are lost.

The arrival rate with i jobs in the system is $(i+1)\lambda$ jobs per second. The service rate with 1 or 2 jobs in the system is μ jobs per second; the service rate with 3 or 4 jobs in the system is 2μ jobs per second. In all three parts of this problem, assume $\lambda = \mu$.

(a) What is the probability that there are more than two jobs in the system?

(b) What is the throughput for the system (the job completion rate of the server), as a function of λ?

(c) How large must λ be to achieve a throughput of 2 jobs/sec?